PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 62 MARCH, 1936 No. 3

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except June and July, at 99–129 North Broadway, Albany, N. Y., by the American Society of Civil Engineers, Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, December 28, 1931, at the Post Office at Albany, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

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NOTE.—The closing dates herein published. are final except when names of prospective discussers are registered for special extension of time.

FOREWORD

By Frank E. Winsor[1], M. Am. Soc. C. E.

The Metropolitan Water District is one of the oldest districts of its kind in the United States, having been established in 1895, following previous investigations which had shown conclusively that many of the communities of which Boston, Mass., is the center could no longer cope with the problem of independent water supplies. Metropolitan Boston is the fourth most populous center in this country, and probably has more independent political units than any other. In addition to 35 cities and towns within a 10-mile radius of the State House in Boston, there are many other communities with populations of from 15 000 to 100 000 within 30 miles. The Metropolitan Water District originally included Boston and thirteen other municipalities to which seven have since been added. Since 1919 this District has been administered by the Metropolitan District Commission. A population of about 1 500 000 is being supplied with Metropolitan water, 15 communities with a population of about 400 000 are entitled legally to join the District, and it may be reasonably expected that other communities with substantial populations will be permitted to join when their needs demand.

From 1895 to 1905 the District acquired the collecting sources and aqueducts owned by the City of Boston and extended the water supply for the District from the Sudbury River sources westward to the Wachusett Reservoir on the Nashua River, at Clinton, Mass., this reservoir having, since completion, furnished the major portion of the water for the District. Water is supplied by wholesale to each community and the community distributes it, fixes its own water rates, and pays its own bills. Payment for the water is based, two-thirds on the metered quantity used and one-third on assessed valuation, all the water being metered.

In 1926, following about eight years of investigation and reports, the Legislature created the Metropolitan District Water Supply Commission, charged mainly with the construction of a major addition to the water supply to be obtained from the Ware and Swift Rivers, which rivers form a part of the drainage area of the Connecticut River. The diversion from these rivers was opposed by the State of Connecticut, and following several years of litigation the Supreme Court of the United States, in March, 1931, fixed certain conditions under which the diversion is being effected with due regard to interstate rights.

The main features of the additional supply, the estimated cost of which was $65 000 000, are: (1) A tunnel, known as Quabbin Aqueduct, 24.6 miles in length, of the horseshoe type, with a waterway of 127 sq ft, extending westerly from Wachusett Reservoir to the Swift River; and (2) a reservoir, known as Quabbin Reservoir, in the valley of the Swift River, which reservoir will flood 25 200 acres to an average depth of about 50 ft, the capacity being 415 000 000 000 gal, or 1 270 000 acre-ft. At about midway of its length the

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[1] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

Quabbin Aqueduct passes 260 ft below the Ware River where diversion is effected through a shaft. After the completion of Quabbin Reservoir, water from the Ware River may be diverted either easterly or westerly and stored in either the Wachusett Reservoir or the Quabbin Reservoir. The Quabbin Aqueduct will be controlled by gates at Wachusett Reservoir, the relative elevations, in feet, above tide-water being: Wachusett, 395; Ware, 656; and Quabbin, 530.

The Ware River was first diverted in March, 1931, and the Quabbin Aqueduct was completed in 1935 sufficiently to permit diversion of the Swift River although such diversion is not contemplated unless the needs of the District require it, until the Quabbin Reservoir is completed probably late in 1939. The construction program has been planned to keep pace with the increasing demands for water and to delay capital expenditure and consequent increase in carrying charges as long as prudently possible.

The paper by Stanley M. Dore, Assoc. M. Am. Soc. C. E., relates mainly to the foundations of the two earth dams of the Quabbin Reservoir, one known as the Dike with an embankment of about 2 500 000 cu yd, and the other known as the Main Dam with an embankment of about 4 000 000 cu yd. The valleys in which these dams are located are both filled with porous glacial drift to a maximum depth of 125 to 130 ft, and it was deemed necessary to build cut-off walls to rock to prevent seepage. The work of building these cut-off walls by the sinking of concrete caissons under air pressure, and the lowering of the ground-water by pumping, to reduce the air pressure required, are described in detail in Mr. Dore's paper. The work on these two dams illustrates the importance of thorough preliminary exploration. The feasibility of lowering the ground-water was demonstrated by exploratory caissons at both sites before bids were invited for building these structures. The uncertainties as to air pressure which would be required to sink the caissons, were eliminated to a great extent, resulting in more dependable and lower bids from contractors and a consequent material saving in the cost of the work to the Commission. So far as the writer is aware, unwatering of the ground on the scale and to the depths accomplished at these two dams is somewhat unique. The paper is a valuable contribution to the knowledge of the flow of ground-water through materials of varying permeability.

The paper by Frank E. Fahlquist, Assoc. M. Am. Soc. C. E., relates mainly to the geological and engineering investigations which were made preliminary to the location of the tunnel and to the tunnel construction. Adequate geological investigations coupled with intelligent sub-surface exploration are essential as a guide to the engineers in the location of such structures as the Quabbin Aqueduct and the dams of Quabbin Reservoir. The location of Quabbin Aqueduct was determined after a most thorough geological investigation supplemented by borings to determine the location and character of ledge rock. Several pitfalls which might have been very costly were thereby avoided, and in the tunnel excavation excellent rock was encountered for the entire 25 miles, with a negligible amount of timber support.

PERMEABILITY DETERMINATIONS, QUABBIN DAMS

By STANLEY M. DORE[2], ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Several methods of determining the permeability of an earth over-burden which forms the foundation for a proposed dam are outlined in this paper. They are based on the assumption that either or both of two general sources of information is available: (1) Dry samples from bore-hole investigations; and (2) the effect of pumping upon the ground-water conditions in that over-burden. These methods were developed in connection with permeability determinations at the sites of the Main Dam and Dike of Quabbin Reservoir,[3] 65 miles west of Boston, Mass., so that in order to describe fully and clearly the methods and their practicable applications much of the text deals with the particular conditions at those sites. The methods were developed primarily for use with materials of glacial origin, in cases where the earth cover is thick and the ground-water surface high in that earth cover, and if carefully applied, they can be adapted to give reasonable determinations under similar conditions elsewhere. Moreover, the principles involved may be adapted for use in cases where the materials are of a different nature, or where some of the other conditions are dissimilar.

INTRODUCTION

A major item in the construction of the Ware River and Swift River developments[3] for increasing the water supply for the Massachusetts Metropolitan Water District, which is comprised of the City of Boston and nineteen neighboring cities and towns, is the Quabbin Reservoir, formed by the construction of two similar earth dams named, for convenience, the Main Dam and the Dike. These two dams have been located in the low portions of the rim of the storage basin, the former being across the valley of the Swift River and the latter across the valley of Beaver Brook, a branch of the Ware River. Both of them are designed as earth dams to be placed by hydraulic methods, the Main Dam being about 2 640 ft long, 160 ft high above the original ground surface, and containing 4 000 000 cu yd of embankment, and the Dike being about 2 140 ft long, 135 ft high above the original ground surface, and containing 2 500 000 cu yd of embankment.

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[2] Assoc. Civ. Engr., Massachusetts Met. Dist. Water Supply Comm., Boston, Mass.

[3] Described by Karl R. Kennison, M. Am. Soc. C. E., in "Boston Metropolitan Water Supply Extension", *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2, and by Frank E. Winsor, M. Am. Soc. C. E., in "Boston's New Water Supply", *Civil Engineering*, June, 1934, p. 283.

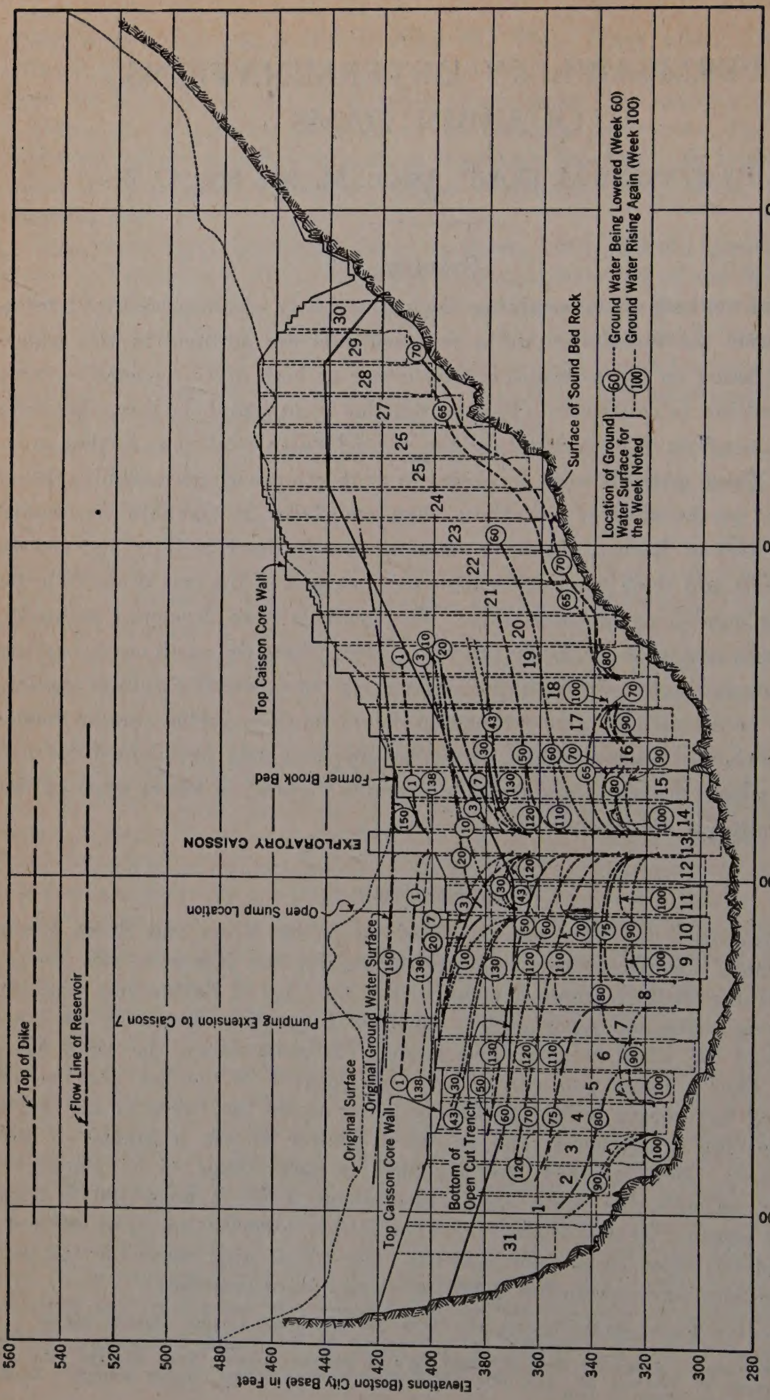
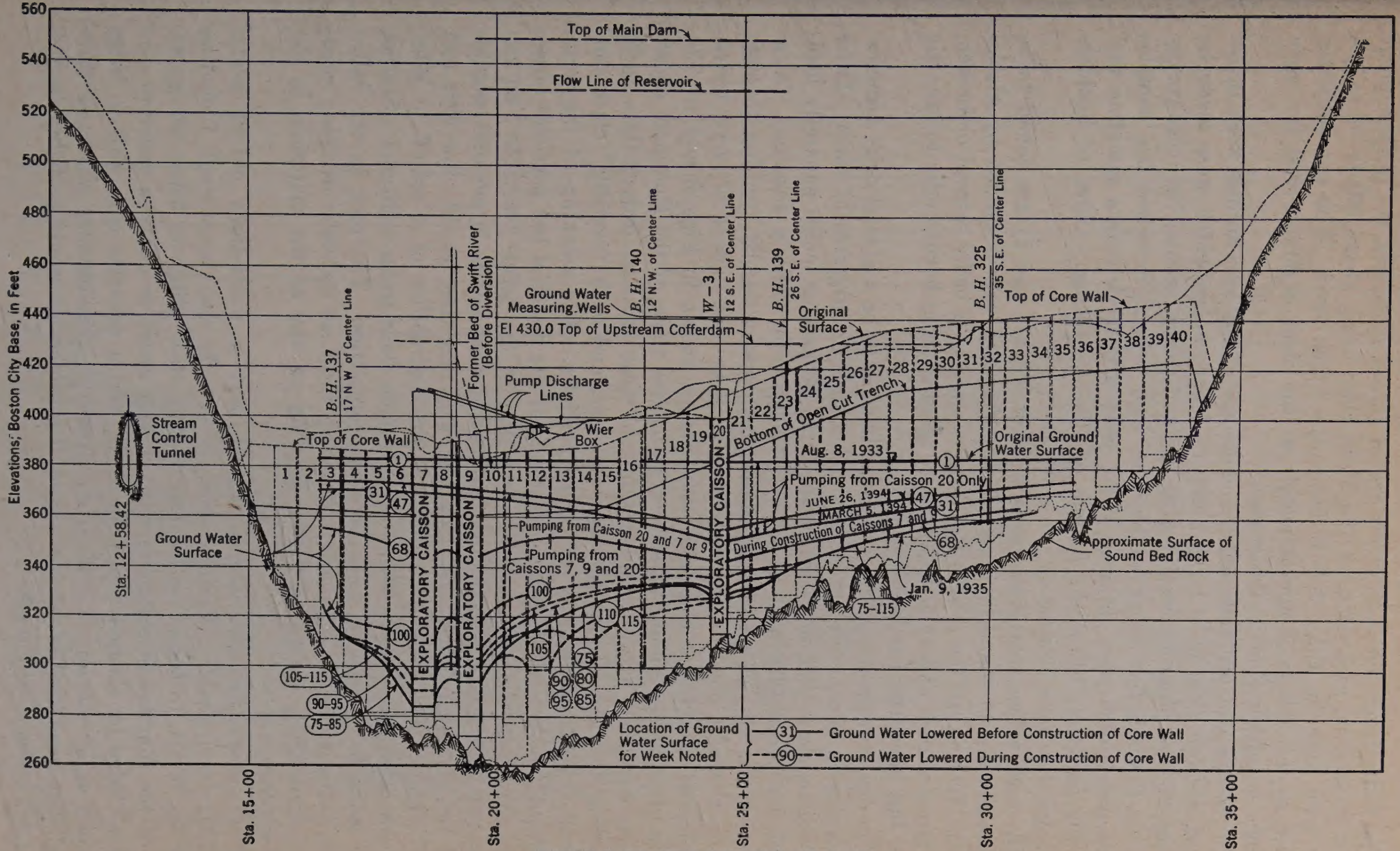


FIG. 1.—CROSS-SECTION OF BEAVER BROOK AT THE SITE OF QUABBIN DIKE


Top of Main Dam
Flow Line of Reservoir
Elevations; Boston City Base, in Feet
560
540
520
500
480
460
440
420
400
380
360
340
320
300
280
260
Stream Control Tunnel
Sta. 12+58.42
Ground Water Surface
B. H. 137 17 N W of Center Line
Top of Core Wall
Former Bed of Swift River (Before Diversion)
Pump Discharge Lines
Wier Box
Ground Water Measuring Wells
B. H. 140 12 N.W. of Center Line
W-3 12 S.E. of Center Line
B. H. 139 26 S.E. of Center Line
B. H. 325 35 S.E. of Center Line
El 430.0 Top of Upstream Cofferdam
Original Surface
Top of Core Wall
Bottom of Open Cut Trench
Original Ground Water Surface
Aug. 8, 1933
Pumping from Caisson 20 Only
JUNE 26, 1934
MARCH 5, 1934
During Construction of Caissons 7 and 9
Pumping from Caisson 20 and 7 or 9
Pumping from Caissons 7, 9 and 20
EXPLORATORY CAISSON
EXPLORATORY CAISSON
EXPLORATORY CAISSON
Approximate Surface of Sound Bed Rock
Jan. 9, 1935
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
① ㉛ ㊼ (68) (100) (105-115) (90-95) (75-85) (105) (110) (115) (75) (80) (85) (90) (95) (75-115)
Location of Ground Water Surface for Week Noted
㉛ Ground Water Lowered Before Construction of Core Wall
(90) Ground Water Lowered During Construction of Core Wall
Sta. 15+00
Sta. 20+00
Sta. 25+00
Sta. 30+00
Sta. 35+00

PROFILE ON CENTER LINE OF DAM
LOOKING UPSTREAM

FIG. 2.—CROSS-SECTION OF SWIFT RIVER VALLEY AT THE SITE OF THE MAIN DAM

The foundations for these dams are in glacial materials, the deepest point in the underlying rock gorge at the Main Dam being about 120 ft below the river level and the Dike, 130 ft below the brook level. The ground-water surface in the valleys is approximately at the river or brook level for some distance on either side of the stream.

The location of bed-rock and the character of the over-burden were investigated at both sites by a large number of bore holes, which were made by driving 2½-in. pipes by wash-boring methods supplemented by core-boring methods through the larger boulders or into ledge. So-called dry samples of the over-burden were obtained about every 5 ft by driving a sampling tool into the materials below the bottom of the pipe-casing. Cores of boulders and ledge also were secured.

The rock valley cross-sections (shown in Figs. 1 and 2) are found to be the usual smooth, rounded, inverted-arch shape of valleys eroded by glacial action. In general, the rock is not decayed; nor is it badly seamed or broken. The types of rock appear to be gneisses, ranging from hornblendic to granitic types, but generally hard and sound.

The dry samples taken show the over-burden to be a glacial drift accumulation, mostly silts, sands, gravels, and boulders or mixtures of them. Later excavations show that these materials lie in pockets or lenses generally horizontal, but absolutely void of any stratification or orderly relation. Occasional streaks or pockets of rock flour were present usually running through the finer sandy deposits, and streaks, pockets, and lenses of very coarse gravel exist elsewhere. Geological sections are shown in Figs. 3 and 4. The names given to the numbered classes, which are those of the Kendorco classification described subsequently herein, in these sections differentiate them according to their relative pervious qualities, and each "variable" class contains a considerable number of larger particles. The meaning of the term, "uniform", is that only a small percentage of the material has particles larger or smaller than the average. The meaning of the term, "variable", on the other hand, is that a considerable number of particles is larger and smaller than the average. These materials are such that they would offer firm and stable support for the earth dams, but the desirability of a cut-off to ledge through these deep over-burdens depends upon their water-tightness. Studies were made to investigate their permeable qualities. The rock in the valley floors, being sound, is assumed to be water-tight, as its water-carrying capacity is negligible compared to the capacity of any of the materials in the over-burdens.

Preliminary investigations indicated that a water-tight cut-off would probably be desirable. Various types of cut-offs were considered, including open-cut trench to ledge, the driving of steel sheet-piling to ledge, the grouting of the foundation materials with cement grout or dobie, a concrete core wall to ledge by sheeted trench methods, and a concrete core wall to ledge by open or by pneumatic caisson construction. Preliminary estimates indicated that some kind of caisson core-wall construction would probably furnish the most economical cut-off.

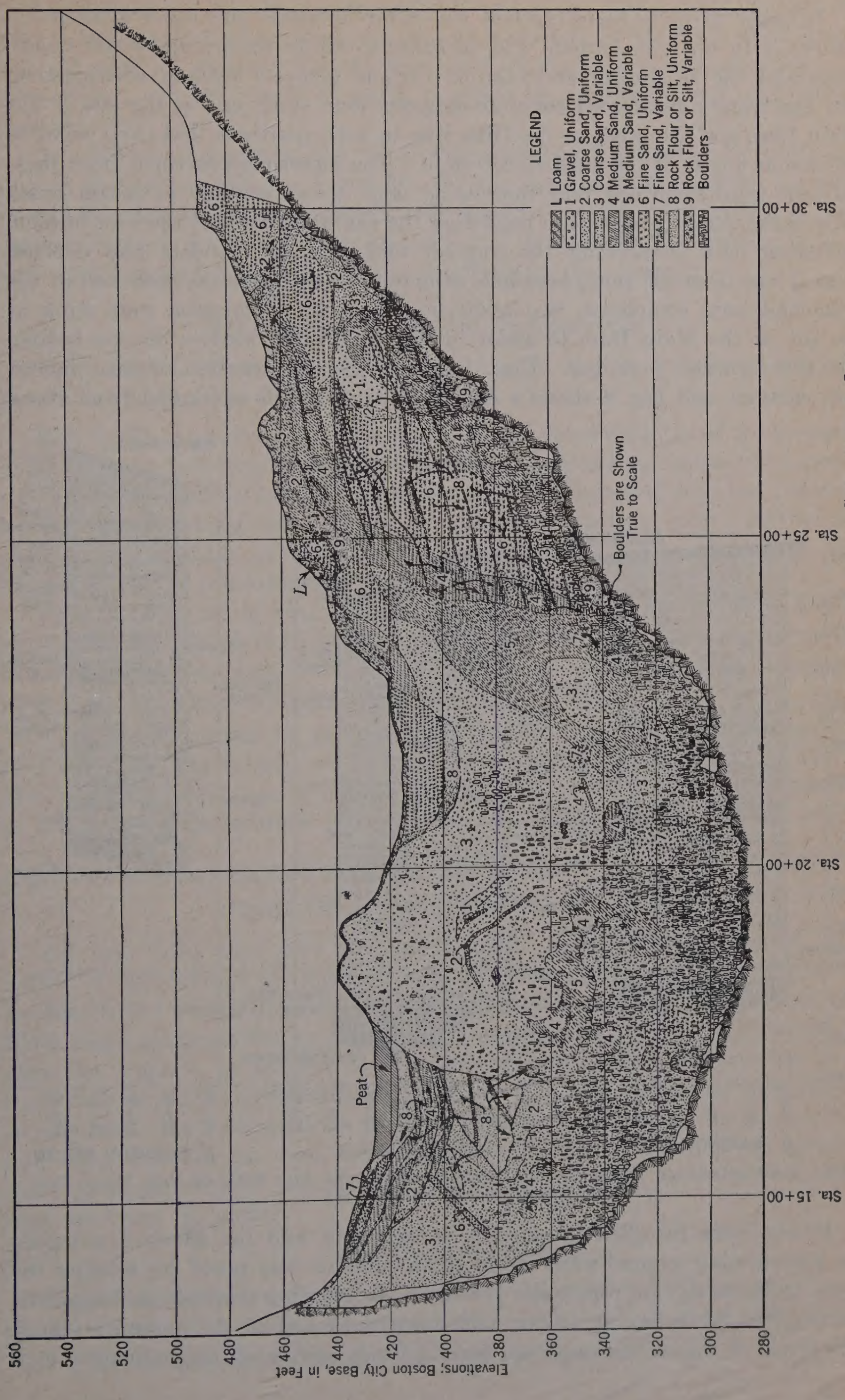


FIG. 3.—GEOLOGICAL SECTION OF BEAVER BROOK AT THE SITE OF THE DIKE

In order to investigate further, the water-tightness and other qualities of materials in the over-burden, and in order to study the ground-water conditions and the feasibility and cost of various types of caisson construction, two exploratory reinforced concrete caissons were sunk, one at the site of the Main Dam and the other at the Dike site, in such positions that each could be utilized as a part of the finished core-wall. The materials excavated from these caissons were examined very thoroughly, and it was possible to obtain much more satisfactory information regarding the characteristics of the over-burden, including data concerning the number and sizes of boulders and cobbles, than it was from the small bore-hole samples. Later, after the core wall of the Dike had been completed, two additional exploratory caissons were sunk at the site of the Main Dam to assist in the exploratory studies of over-burden and ground-water pumping. Figs. 1 and 2 show the location of the exploratory caissons and Fig. 4 shows a chart of the materials excavated from them.

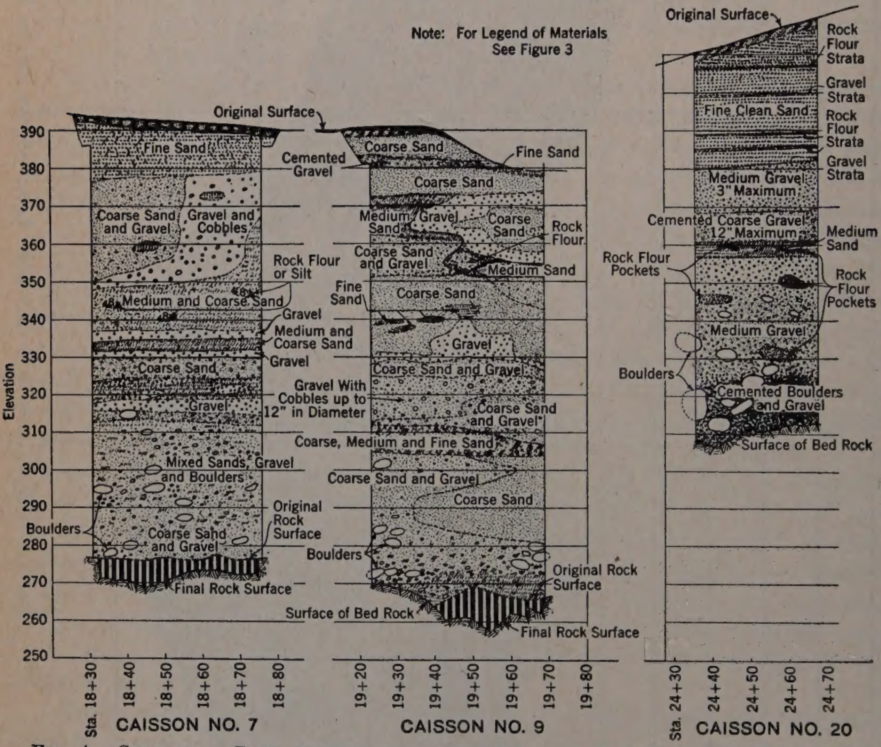


FIG. 4.—GEOLOGICAL DATA, EXPLORATORY CAISSONS, AT THE SITE OF THE MAIN DAM (FOR LEGEND SEE FIG. 3)

Pumps were installed in caissons at each site, and the effect of pumping the surrounding ground-water from these caissons was noted by reading the water levels in 2½-in. pipe wells driven by wash-boring methods at designated locations in the valley up stream and down stream from the pumping plants. The levels of the water were read several times each week by lowering a cupshaped washer into the wells. In addition to aiding in the determination of the effective water-tightness of the glacial materials in the valley over-burden, the effect of pumping also demonstrated the feasibility of unwatering the over-burden, and the extent to which it would be practicable and economical so to do.

As used in this paper, the term, "permeability", measured in units of million gallons per acre per day at 50° F for a slope of unity, denotes the rate at which soils, sands, and gravels will transmit water. "Porosity" is the expression used to denote the percentage of a given volume composed of air and water pockets or "voids" in the materials in that volume.

Characteristics of the Over-Burdens

The permeability of the over-burden at each site is determined by two general methods given in Parts I and II, as follows:

Part I: The classification of bore samples and the estimation of the permeability of each class from laboratory analyses. There are two methods of using the laboratory data to make partly independent estimates of the class permeabilities: (*a*) By computation from the mechanical analysis curves derived by the sieving of class samples; and (*b*) by testing in the laboratory the permeabilities of representative class samples.

Part II: The use of the pumping rates, the resulting ground-water levels, and other pertinent data in connection with the lowering of the ground-water at each site by pumping to determine the average effective permeable quality of the over-burden as a whole.

The determinations by all the methods described are based on the law of Darcy[4] that the velocity of flow of water through a column of soil varies directly as the difference in pressure on the ends of the column and inversely as the length of the column.

PART I.—PERMEABILITY DETERMINATIONS FROM CLASSIFICATION OF BORE-HOLE SAMPLES

All bore holes that were properly situated in the deeper parts of each valley in the vicinity of each site were taken as representative of the over-burden in the foundations at that site. Thirty deep holes were used at the Main Dam and thirty-five at the Dike. They were well distributed horizontally over the areas to be covered by the embankments. The dry samples from the bore-holes were representative of the materials from which they were taken, as apparently the finer particles were not washed from them by the wash water and other fines did not wash into them from the work above. These samples were examined visually and classified according to their "look and feel" into nine arbitrary divisions.

The Classification

This classification is named the "Kendorco Classification" and was developed for use on this project to denote and describe roughly the general kinds of the glacial soils in the vicinity. Of course, there are no distinct divisions between the classes, but attention is called to the general differences in physical characteristics given in Table 1 and to the arbitrary limits set for the classes as shown in Fig. 5. The types of material in Table 1 may be further described, as follows:

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[4] "Les fontaines publiques de la ville de Dijon", by H. Darcy, 1856.

TABLE 1.—PHYSICAL CHARACTERISTICS OF SOIL SAMPLES

Class	Name of class	AVERAGE SIZES OF A TYPICAL SAMPLE, IN MILLIMETERS			Computed permeability, in million gallons daily per acre, at 50° F for slope of unity	AVERAGE OF LABORATORY DETERMINATIONS		
		Range of bulk of the sample	Effective size; 10% of the sample	Smaller size		Porosity (percentages)	Permeability in Million Gallons Daily per Acre, at 50° F (in Natural State)	
							Main Dam	Dike
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Clean gravel......	10.0 to 2.0	1.50	4 %<0.3	540	32	540	....
2	Coarse sand, uniform...........	1.5 to 0.3	0.31	4 %<0.15	70	46	110	....
3	Coarse sand, variable...........	Variable	0.35	4 %<0.15	12	25	15	12
4	Medium sand, uniform...........	0.7 to 0.2	0.15	4 %<0.07	20	49	38	14
5	Medium sand, variable...........	Variable	0.175	4 %<0.07	4	27	3.0	3.5
6	Fine sand, uniform	0.3 to 0.1	0.065	6 %<0.03	2.5	43	2.0	1.5
7	Fine sand, variable	Variable	0.075	6 %<0.03	0.8	27	0.8	0.7
8	Flour, uniform....	0.1 to 0.02	0.013	8 %<0.01	0.1	41	0.15	0.2
9	Flour, variable....	Variable	0.025	8 %<0.01	<0.1	24	<0.1	<0.1

Class Description

1.—A material that contains large particles of gravel and very coarse sand, and very few fines.

2.—A uniform coarse sand that contains only a few fines and no gravel or stones.

3.—A material of which the part controlling the permeability is coarse sand, but in which there are a few fines, some gravel, or both.

4.—A uniform medium sand containing but few fines and no gravel or stone.

5.—A material of which the part controlling the permeability is medium, but in which there are finer and coarser sands and possibly gravel or stones.

6.—A uniform fine sand containing only a small percentage of larger or smaller grains than the average.

7.—A material of which the part controlling the permeability is a fine sand but in which there is a large percentage of coarser materials, possibly gravel or stone.

8.—A rock flour or silt containing only a small percentage of grains larger than the average.

9.—A material of which there is a large quantity of rock flour or silt but in which there is a large percentage of coarser materials, possibly gravel or stone.

The values in Column (6), Table 1, were computed for the tested porosity shown in Column (7). The average of all determinations for porosity was 35%; the straight average porosity of the nine classes was also 35%;

the weighted mean porosity for material at the Main Dam was 37% and at the Dike, 39%; and the mean effective porosity (obtained from the permeability of the over-burden as a whole) at the Main Dam was 33%, and at the Dike, 37 per cent.

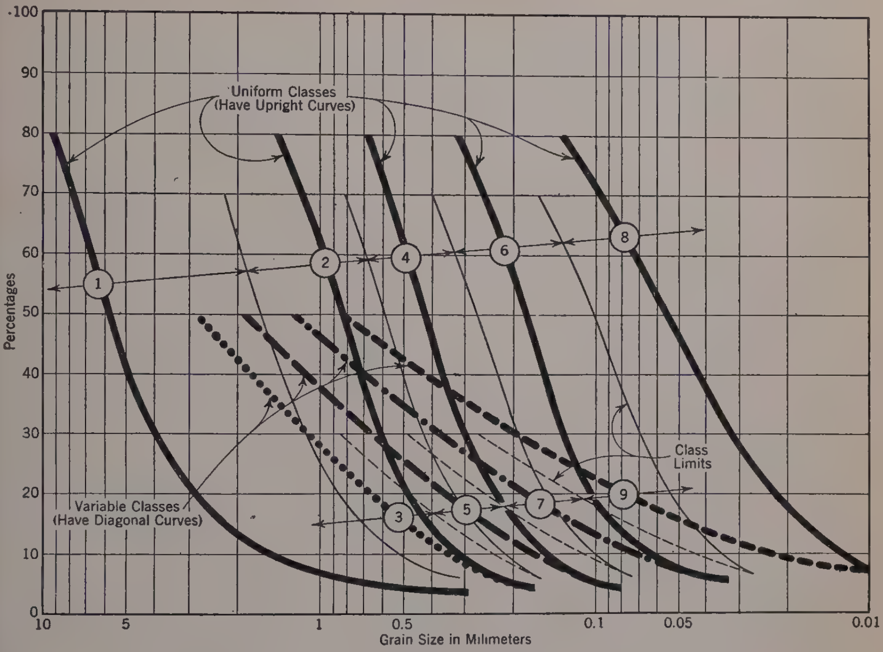

FIG. 5.—DEFINITIONS OF LIMITS, KENDORCA SOIL CLASSIFICATION

When the classification of all dry samples was complete, five were chosen carefully from each class at each site to show the range of all the samples of that general class. In choosing these samples, three were selected to represent the average of the entire classification, a fourth the coarser extreme, and the fifth the finer extreme. Sieve analyses were made on the five samples of each class (forty-five samples for each site). Such data were used as the basis for the diagram of class curves shown in Fig. 5. This system of general classification is easy to apply and fairly accurate in its results, even when applied by those relatively inexperienced with it, although there are always certain to be differences in opinion regarding border-line samples.

CLASSIFICATION OF THE OVER-BURDENS

Materials in the over-burdens at the Main Dam and at the Dike were assigned in both plan and elevation to the nine classes. This assignment was made according to the number of linear feet of the bore holes of each class of material encountered, each hole being weighted according to its plan location so that it represents its share horizontally of the entire valley.

PERMEABILITY OF THE CLASSES (COMPUTED FROM SIEVE ANALYSES)

The permeability of each class of material was computed from the mechanical analyses curves representing the averages of those classes by use of the formula introduced by Charles S. Slichter.[5] In applying Slichter's formula for determining ground-water flow, it has been the practice of many engineers to use erroneously in place of the "mean diameter" defined by Slichter, the effective size (10% size) defined by the late Allen Hazen,[6] M. Am. Soc. C. E., although the two, by definition, are different. Stearns,[7] King,[7] Melcher,[7] Slichter, and others, failed to find any consistent agreement between the 10% sieve size and the coefficient of permeability for most types of mixed sands and gravels. It is believed that the 10% size used by Hazen is applicable only to the filter sands with which he worked, and that for "variable" glacial materials the controlling size depends upon local characteristics of the materials, such as the shape of grains and the combination of sizes.

If permeability of a material or of an over-burden is to be computed by this formula it is recommended that, if practicable, many permeability tests be made on samples from that over-burden to determine the size, from sieve analyses curves, that will give dependable results when used instead of the mean diameter in Slichter's formula, otherwise, the determination may be very misleading.

Several hundred permeability tests were made on materials from overburdens of the Main Dam and the Dike, at known porosities and temperatures, and a sieve analysis curve of each of the samples was plotted. The size producing the tested permeability was obtained by substituting the permeability result in Slichter's formula in each case. The percentage grain size was chosen by inspecting the sieve analysis curve of that sample. From the large number of tests and inspections made, the size to be substituted in the formula to produce the tested permeabilities varied for these sites from the 5% to the 26% size, although the majority of the results were confined to closer limits, between the 8% and the 15% size. For these sands and gravels, the reasonable mean of all results seems to be the 12% size.

In order to compute the permeability of any material from its mechanical analyses curve, its porosity must be known. A large number of field tests were conducted at many locations in the excavations at the sites of the Main Dam and the Dike to determine the porosity of the materials in the overburden in general and particularly to determine the proper porosity that will be applicable to each classification. The results of these tests are shown in Table 1.

The temperature of the ground-water for any locality must also be known, and for these locations it was approximately 50° F. Fifty-six observations of ground-water flowing into the Quabbin Tunnel, which runs about parallel to the sites and about 6 miles to the north, were made at locations 80 to 600 ft below the ground surface and at intervals in a length of 10 miles of the tunnel. The average of all observations was 51° F, the lowest being 48° F, and the highest, 53½° F. The water pumped from the caissons at both the Main Dam and the Dike maintained a fairly constant temperature very close to 50° F, throughout the year (between 48° and 52°). Therefore, 50° F would be the proper value to use, especially as it is approximately the mean annual temperature for the climate of this region.

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[5] *Water Supply Papers Nos. 67* and *140*, U. S. Geological Survey.

[6] Rept., Mass. State Board of Health, by Allen Hazen, 1892.

[7] See *Water Supply Paper 596-F*, by N. D. Stearns.

The permeabilities of the nine classes were then computed for the tested porosities at a temperature of 50° F, using Slichter's formula and the 12% size from the theoretical curve for each class as the mean diameter (such curve representing the average of materials in that class). The results of such computations are shown in Table 1.

TESTED PERMEABILITY OF THE CLASSES

A large number of samples of materials belonging to each of the nine classes was taken directly from open-cut, or caisson, excavations in the overburden of the Main Dam and Dike and were tested for permeability. Care was taken to select samples representing the class averages. For the most part these were small can tests made at 68° F, the material being packed by hand to known porosities with as little mixing as possible.

Several permeability tests of each sample were made at different porosities, and a permeability-porosity curve for that sample was plotted. From this curve, the permeability was read for the average porosity of that class previously determined, and the result was corrected[8] to a temperature at 50° F. The permeabilities thus obtained are shown in Table 1.

PERMEABILITY OF THE OVER-BURDEN FROM THE CLASSIFICATION PERMEABILITIES

The over-burdens were composed of heterogeneous glacial materials, in the valley inverts being forms of modified drift. The various classes of materials were not laid in strata or layers or with any order at all, but seem to be deposited in random lenses or pockets. In view of this lack of order, the question arises (assuming that the permeabilities of the respective classes and quantities of each class of material in the over-burden are satisfactorily computed or known) as to how best to obtain the permeability of the over-burden as a whole. For instance, the maximum total permeability of a given quantity made up of an equal quantity of each class would be for a condition in which the amounts of each class are laid in horizontal layers parallel to the flow, and the minimum would be for a condition in which the quantities are placed in vertical layers perpendicular to the flow. In the former case, herein termed "in parallel", the direct sum of the quantities carried by each layer will give the total carried by the mass, and from this the unit permeability of the mass can be computed. The average effective unit permeability of the whole mass, for that case, will be equal to the unit permeabilities of the component parts averaged and weighted in accordance with the relative quantities of the parts. In the latter case, herein called "in series", the average effective permeability of the mass is only a fraction of the "parallel" quantity, as the finer layers control the amount of flow through the coarser layers and prevent them from carrying larger quantities. However, any contact or linking of coarser classes, although by links of relatively smaller volumes, will so increase the effective permeability of the entire mass over that obtained by the "series" method that such permeability will be found to be a very large percentage of the "parallel" method, and except for extremely small linking volumes, this percentage is very close to 100. In random glacial deposits there is usually no uniform tendency for complete separation of the coarser parts by the finer. On the other hand, during the period of deposition and in the following washing periods, water tends to open passages of flow between pockets, if they do not already exist. Therefore, it is believed that approximately close results can be obtained for glacial materials if the average effective permeability is computed from the unit class permeabilities by the "parallel" combination. Such an approximation is on the safe side as far as percolation or seepage quantities are concerned, and probably is not far from being representative of true conditions in the over-burden.

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[8] Correction factors are given in "Smithsonian Physical Tables—Viscosity of Water in Centipoises *vs.* Temperature Variation", the permeability being varied inversely with the viscosity.

As the permeability of the over-burden as a whole is so dependent upon the manner in which the permeabilities of the various classes are combined, too much emphasis cannot be placed upon the importance of obtaining as much information as practicable regarding the existence and manner of occurrence of the various classes in the over-burden.

In the case of these sites, investigations of existing excavations in the open and in the caissons, of the bore-hole data, and of other geological data, indicate that probably numerous interconnections and intermeshings of the coarser lenses, streaks and pockets with the finer exist, and that there is no tendency toward segregation of lenses in such manner that the finer would separate and isolate the coarser. Thus, the resulting permeabilities at the sites of the Main Dam and Dike are estimated, using 100% of the results obtained by the "parallel" method.

Although the results of these classification methods may be open to criticism owing to the many arbitrary assumptions necessary to obtain them, it is believed that such results are adequate in this case and will be in similar cases, as the results for both these sites are confirmed by the determinations of permeability made by the pumping methods described subsequently herein, under Part II. The pumping results do not depend upon arbitrary assumptions or decisions and, therefore, this confirmation of results may be used as an independent check of those made for the bore-hole classification methods. The over-burdens at these sites have no conditions which are unusual or uncommon to glacial-formed valley deposits and, consequently, there seems to be no reason why such similar methods will not give equally good results elsewhere.

Although such methods are approximations in many respects and although the computed results cannot be treated with too much exactness, they can be used to obtain more comprehensive opinions of the permeability of any extensive glacial materials than any other methods known to the writer except the pumping methods described subsequently. Any determination should be used within the limits of accuracy considered reasonable, and for permeability of an over-burden these limits are quite elastic. For instance, limits between 50% and 200% of a result computed by these methods may not be unreasonable, although the writer believes that the results for these sites and for other similar sites may be computed within much closer limits. However, even a result with wide elastic limits is much better than no determination and to the engineer considering seepage possibilities, such limits are usually close enough for his purposes.

PART II.—PERMEABILITY DETERMINATIONS FROM EFFECTS OF GROUND-WATER PUMPING OPERATIONS

Pumping at the Dike Site

For examining the materials in the over-burden and for experimenting with pumping of ground-water, Caisson 13 was sunk at the Dike in a location where the ground-water surface was practically at the original surface, which is about 130 ft above ledge (see Fig. 1). During the sinking, if material below the ground-water level was being excavated under pneumatic pressures, it was impracticable to lower the ground-water outside the caisson by pumping from within for two reasons: (1) The presence of air in the pores of the surrounding soils materially reduced the volume of the pores available for the flow of ground-water toward the caisson; and (2) the use of air (probably because it tended to break up and separate the fines) seemed to accelerate the flow of the fine particles near the pumping intakes into the intakes, and thereby clogging them quickly. As a head of water of 130 ft would give unreasonably high pressures for pneumatic work (50 lb per sq in. being the legal limit in Massachusetts), it was necessary to lower the ground-water level near the caisson in order to sink it completely.

When the caisson had been sunk 63 ft below the brook level, sinking operations were suspended and pumps were installed in the working chamber of the caisson in order that the ground-water level might be lowered to provide for the sinking of an open external sump, to be used for pumping while the caisson was being sunk. The water was pumped from the working chamber of the caisson at that depth at an average rate of about 1 600 gal per min. for a period of about seven weeks. The sump, about 26 ft square, of wooden sheet-piling, was sunk at a location about 100 ft from the caisson, to below the lowered water level, the depth being about 50 ft below the brook level. In that period of seven weeks the water level had been lowered to a depth of about 38 ft below the brook level, the sump being completed in the wet below that depth by additional pumping at a small rate and by driving the sheeting as far ahead of the excavation as boulders would permit.

The pumps were then removed from the caisson and installed in the open sump until after the caisson was completed. Pumping intakes were built under the cutting-edge in the underpinning foundations of the caisson. During this period of about thirteen weeks the water drawn from the open sump averaged about 1 100 gal per min; in the sump, the minimum water level was about 45 ft below, and at the caisson about 30 ft below, the brook level. The highest air pressure used in sealing the caisson to ledge was about 48 lb per sq in. Pumps were then installed in the working chamber of the caisson utilizing the intakes previously built and the ground-water was drawn from the caisson and the sump jointly for a period of five months at an average rate of about 1 600 gal per min.

At the start of the core-wall construction (January, 1933), greater pumping capacity was installed in Caisson 13 and pumping from the open sump was abandoned. Pumping continued from that caisson for about four months at the increased rate of about 3 200 gal per min., at which time Caisson 18 (see Fig. 1) had been sunk to a depth near ledge. Pumps were installed in the working chamber of Caisson 18 to supplement those of Caisson 13 and water was drawn at an average of about 3 000 gal per min from both caissons (about 2 600 gal from Caisson 13 and 400 gal from Caisson 18). This work was continued for ten weeks more.

During the sinking of Caisson 13, to aid the pumping operations, Beaver Brook was carried past the site in a wooden flume extending from a point 200 ft up stream from the center line of the Dike to a point about 160 ft down stream. During the construction of the core-wall this brook was dammed off about 500 ft up stream, the pumping discharge being deposited 400 ft down stream. The subsequent pumping operations dried up the brook up stream and lowered the level of Morton Pond, which is about 1 500 ft up stream, a maximum of about 9 ft (see Fig. 6).

Other caissons were sunk below the ground-water gradient and more water was pumped from the working chambers at intervals when no material was being excavated. When they were sealed to ledge many of the caissons were equipped with pumping intakes which were also used to lower the ground-water while sinking the other caissons. Ground-water was pumped in this manner from Caissons 18, 16, 8, 6, 4, 10, and 7, at the intervals and in the quantities shown in Table 2(*a*).

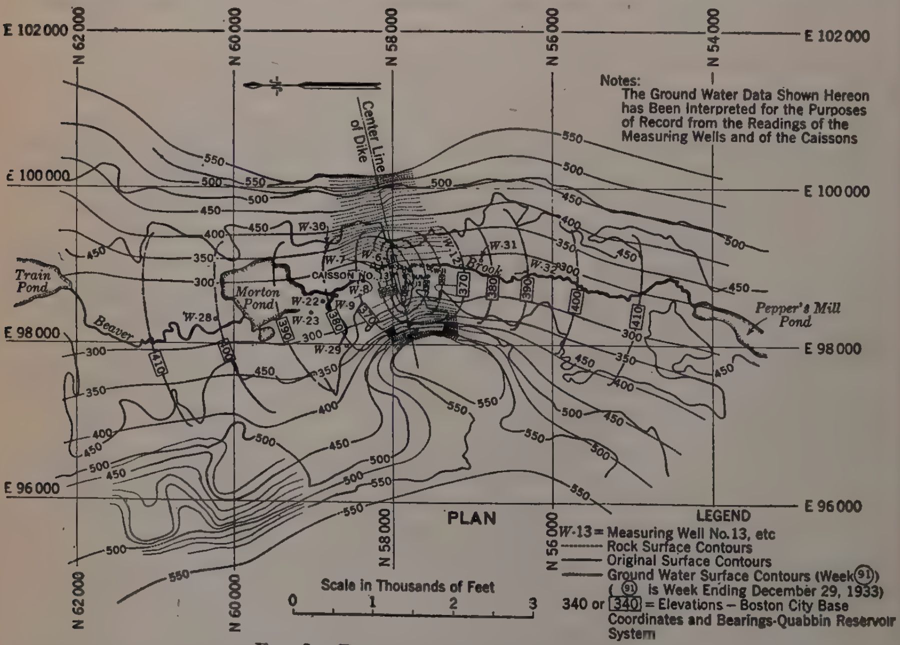
E 102 000
N 62 000
N 60 000
N 58 000
N 56 000
N 54 000
E 102 000
Center Line of Dike
Notes:
The Ground Water Data Shown Hereon has Been Interpreted for the Purposes of Record from the Readings of the Measuring Wells and of the Caissons
E 100 000
E 100 000
550
500
450
400
350
300
W-30
W-7
W-6
W-31
W-32
CAISSON NO. 13
W-8
W-9
W-22
W-23
W-28
W-29
Brook
Beaver
Train Pond
Morton Pond
Pepper's Mill Pond
370
380
390
400
410
E 98 000
E 98 000
E 96 000
E 96 000
PLAN
Scale in Thousands of Feet
0
1
2
3
LEGEND
W-13 = Measuring Well No. 13, etc
Rock Surface Contours
Original Surface Contours
Ground Water Surface Contours (Week ㉛)
(91 is Week Ending December 29, 1933)
340 or 340 = Elevations – Boston City Base
Coordinates and Bearings-Quabbin Reservoir System

FIG. 6.—PLAN AT THE SITE OF THE DIKE

TABLE 2.—OBSERVED RAINFALL AND PUMPING RATES

Week No.	(a) AT THE DIKE					(b) AT THE MAIN DAM			Elevation of Water Surface, Swift River, in Feet	
	Week ending	Total rainfall, in inches	Average pumping rate, in gallons per minute	Location of pumping; Caisson*:	Elevation of water surface, Morton Pond, in feet	Week ending	Average pumping rate, in gallons per minute	Location of pumping; Caisson†:	Up stream	Down stream
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	4-23-32	....	995	13	416.97	8-11-33	.....	..	......	......
2	4-28-32	0.38	1 275	13	416.87	8-18-33	1 158	20	......	......
3	5- 5-32	0.80	1 953	13	416.86	8-25-33	2 220	20	383.47	381.57
4	5-11-32	0.39	2 016	13	416.74	9- 1-33	2 224	20	383.24	381,39
5	5-17-32	....	1 820	13	416.61	9- 8-33	2 190	20	383.94	381.89
6	5-23-32	....	1 785	13	416.51	9-15-33	2 269	20	383.40	381.52
7	5-27-32	0.30	1 655	13	416.47	9-22-33	2 245	20	388.12	385.69
8	6- 2-32	0.14	1 332	*S*	416.47	9-29-33	2 262	20	385 20	383 54
9	6- 8-32	0.59	1 370	*S*	416.46	10- 6-33	2 260	20	384.30	382.94
10	6-14-32	0.10	1 360	*S*	416.39	10-13-33	2 260	20	384.79	383.26
11	6-20-32	1.47	1 268	*S*	416.51	10-20-33	2 260	20	384.48	383.04
12	6-29-32	0.43	1 246	*S*	416.42	10-27-33	2 257	20	385.22	383.54
13	7- 5-32	2.03	1 016	*S*	416.48	11- 3-33	2 249	20	385.11	383.46
14	7-11-32	0.77	1 022	*S*	416.46	11-10-33	2 240	20	384.34	382.91
15	7-17-32	0.06	1 032	*S*	416.33	11-17-33	2 210	20	384.38	382.95
16	7-23-32	0.34	1 012	*S*	416.26	11-24-33	1 844	20	384.17	382.81
17	7-29-32	0.51	984	*S*	416.25	12- 1-33	1 796	20	384.40	382.96
18	8- 4-32	1.74	950	*S*	416.27	12- 8-33	1 894	20	384.34	382.92
19	8-10-32	0.38	827	*S*	416.36	12-15-33	1 803	20	384.53	382.82
20	8-16-32	0.25	837	*S*	416.26	12-22-33	2 143	20	......	382.74
21	8-22-32	0.76	1 000	13, *S*	416.19	12-29-33	2 159	20	......	382.5±
22	8-28-32	0.24	1 332	13, *S*	416.13	1- 5-34	2 170	20	......	382.65
23	9- 6-32	1.75	1 189‡	13, *S*	416.05	1-12-34	2 210	20	385.27	383.28
24	9-15-32	....	1 144	13, *S*	415.92	1-19-34	2 087	20	384.59	383.03
25	9-23-32	2.43	2 137	13, *S*	415.90	1-26-34	.....	..	......	......
26	9-30-32	1.37	1 898	13, *S*	415.74	2- 2-34	2 080	20	384.57	......
27	10- 7-32	1.41	1 895	13, *S*	415.55	2- 9-34	2 080	20	384.00	......
28	10-14-32	0.09	1 860	13, *S*	415.32	2-16-34	2 074	20	383.70	......
29	10-21-32	1.81	1 855	13, *S*	415.15	2-23-34	2 070	20	383.60	......
30	10-28-32	0.70	1 855	13, *S*	415.14	3- 2-34	2 058	20	383.64	......
31	11- 4-32	0.80	1 829	13, *S*	415.07	3- 9-34	2 050	20	386.69	384.91
32	11-11-32	2.14	1 778	13, *S*	415.17	3-16-34	2 021	20	385.28	383.62
33	11-18-32	0.54	1 748	13, *S*	415.58	3-23-34	1 940	20	385.94	383.96
34	11-25-32	0.89	1 740	13, *S*	415.74	3-30-34	1 999	20	386.71	384.56
35	12- 2-32	....	1 726	13, *S*	415.59	4- 6-34	2 042	20	388.30	385.83
36	12- 9-32	0.28	1 721	13, *S*	415.36	4-13-34	2 106	20	386.78	384.74
37	12-16-32	0.54	1 723	13, *S*	415.23	4-20-34	2 120	20	387.98	385.61
38	12-23-32	0.33	1 727	13, *S*	414.93	4-27-34	2 036	20	386.39	384.44
39	12-30-32	0.74	1 719	13, *S*	415.06	5- 4-34	2 009	20	385.36	383.61
40	1- 6-33	....	1 694	13, *S*	415.32	5-11-34	1 999	20	386.21	384.37
41	1-13-33	0.97	1 699	13, *S*	415.27	5-18-34	1 901	20	385.88	384.00
42	1-20-33	0.86	1 817	13, *S*	415.33	5-25-34	1 900	20	384.84	383.30
43	1-27-33	0.59	1 334	13, *S*	415.68	6- 1-34	1 900	20	384.63	383.15
44	2- 3-33	0.22	1 262	13	415.82	6- 8-34	1 983	20	384.28	382.88
45	2-10-33	1.48	1 517	13	415.87	6-15-34	1 917	20	383.95	382.63
46	2-17-33	0.54	2 353	13	416.19	6-22-34	1 933	20	384.53	382.93
47	2-24-33	1.01	2 661	13	416.34	6-29-34	1 335	9, 20	384.57	383.01
48	3- 3-33	1.23	2 583	13	416.64	7- 6-34	2 814	9, 20	383.47	382.27
49	3-10-33	1.09	2 558	13	416.69	7-13-34	2 583	7, 9, 20	383.20	382.10
50	3-17-33	0.85	2 909	13	416.91	7-20-34	2 846	9, 20	382.85	381.94
51	3-24-33	2.02	3 108	13	417.40	7-27-34	3 213	7, 9, 20	382.74	381.92
52	3-31-33	0.84	3 206	13	417.95	8- 3-34	3 133	9, 20	383.74	382.49
53	4- 7-33	1.48	3 249	13	418.61	8-10-34	3 030	9, 20	382.91	382.02
54	4-14-33	3.76	3 181	13	419.00	8-17-34	2 825	9, 20	382.72	381.92
55	4-21-33	1.48	3 200	13	419.14	8-24-34	3 428	7, 9, 20	382.79	381.95
56	4-28-33	0.59	3 189	13	418.46	8-31-34	3 979	7, 20	382.97	382.03
57	5- 5-33	0.52	3 228	13	417.76	9- 7-34	4 046	7, 20	382.78	381.93
58	5-12-33	0.35	3 214	13	418.00	9-14-34	2 364	7, 9, 20	384.34	383.06
59	5-19-33	....	3 200	13	417.10	9-21-34	3 643	7, 9, 20	385.41	383.62
60	5-26-33	0.98	3 337	13, 18	417.14	9-28-34	4 258	7, 20	383.97	382.70

* See Fig. 1, *S* denotes "Open Sump". † See Fig. 2. ‡ On one day there were no records.

TABLE 2.—*Continued*

Week No. (1)	(a) AT THE DIKE					(b) AT THE MAIN DAM				
	Week ending (2)	Total rain-fall, in inches (3)	Average pumping rate, in gallons per minute (4)	Location of pumping; Caisson*: (5)	Elevation of water surface, Morton Pond, in feet (6)	Week ending (7)	Average pumping rate, in gallons per minute (8)	Location of pumping; Caisson†: (9)	Elevation of Water Surface, Swift River, in Feet	
									Up stream (10)	Down stream (11)
61	6- 2-33	0.52	3 428	13, 18	417.03	10- 5-34	4 027	7, 20	384.92	383.41
62	6- 9-33	1.70	3 336	13, 18	416.83	10-12-34	4 172	7, 20	384.46	383.04
63	6-16-33	0.11	3 171	13, 18	416.52	10-19-34	3 861	7, 20	383.84	382.56
64	6-23-33	0.76	2 976	13, 18	416.14	10-26-34	2 378	7, 9, 20	383.82	382.60
65	6-30-33	0.06	3 138	13, 18	415.61	11- 2-34	2 332	9, 20	384.40	383.01
66	7- 7-33	0.48	3 041	13, 18	415.07	11- 9-34	2 546	9, 20	385.26	383.63
67	7-14-33	0.45	2 964	13, 18	414.30	11-16-34	2 236	9, 20	384.75	383.11
68	7-21-33	0.17	2 877	13, 18	413.69	11-23-34	2 637	9, 20	384.17	382.80
69	7-28-33	1.60	2 952	13, 18	413.19	11-30-34	5 271	7, 9, 20	384.25	382.84
70	8- 4-33	0.97	2 948	13, 16, 18	412.55	12- 7-34	5 015	7, 9, 20	386.48	384.42
71	8-11-33	0.30	3 182	8, 13, 16	411.85	12-14-34	4 617	7, 9, 20	384.50	383.07
72	8-18-33	2.35	3 079	6, 13, 16	411.29	12-21-34	4 319	7, 9, 20	384.25	382.98
73	8-25-33	2.17	3 253	8, 6, 13, 16	410.80	12-28-34	4 223	7, 9, 20	384.64	383.04
74	9- 1-33	0.30	3 352	8, 13, 16	410.38	1- 4-35	4 098	7, 9, 20	384.24	382.88
75	9- 8-33	3.99	3 230	8, 13, 16	410.01	1-11-35	3 772	7, 9, 20	385.82	384.00
76	9-15-33	3.60	3 028	8, 13, 16	409.71	1-18-35	3 662	7, 9, 20	386.27	384.33
77	9-22-33	2.69	3 252	8, 13, 16	412.17	1-25-35	3 590	7, 9, 20	384.93	383.38
78	9-29-33	0.22	3 446	6, 8, 16, 13	412.17	2- 1-35	3 483	7, 9, 20	384.53	383.07
79	10- 6-33	1.43	3 094	6, 16, 13	411.29	2- 8-35	3 430	7, 9, 20	384.22	382.92
80	10-13-33	0.43	3 053	6, 16, 13	410.94	2-15-35	3 382	7, 9, 20	384.12	382.82
81	10-20-33	1.01	3 218	6, 16, 13	410.45	2-22-35	3 359	7, 9, 20	384.96	383.42
82	10-27-33	1.80	3 062	6, 10, 13, 16	410.31	3- 1-35	3 341	7, 9, 20	385.05	383.48
83	11- 3-33	0.03	3 250	6, 10, 13, 16	410.52	3- 8-35	3 264	7, 9, 20	385.43	383.68
84	11-10-33	0.35	2 400	6, 10, 13, 16	410.24	3-15-35	3 292	7, 9, 20	386.38	384.35
85	11-17-33	0.64	3 123	8, 13, 16	410.06	3-22-35	3 363	7, 9, 20	386.75	384.62
86	11-24-33	0 02	3 063	8, 13, 16	409.73	3-29-35	3 425	7, 9, 20	386.17	384.15
87	12- 1-33	0.89	3 290	8, 13, 16	409.59	4- 5-35	3 434	7, 9, 20	386.05	384.07
88	12- 8-33	0.63	3 321	4, 8, 13, 16	409.47	4-12-35	3 467	7, 9, 20	385.50	383.67
89	12-15-33	0.86	3 145	4, 8, 13, 16	409.26	4-19-35	3 436	7, 9, 20	386.82	384.64
90	12-22-33	1.22	2 842	4, 8, 13, 16	408.72	4-26-35	3 469	7, 9, 20	385.68	383.85
91	12-29-33	0.71	2 742	4, 8, 13, 16	408.77	5- 3-35	3 512	7, 9, 20	384.97	383.32
92	1- 5-34	0.75	2 724	4, 8, 10, 13, 16	408.52	5-10-35	3 509	7, 9, 20	385.63	383.78
93	1-12-34	1.18	2 739	4, 8, 10, 13, 16	408.94	5-17-35	3 533	7, 9, 20	385.12	383.48
94	1-19-34	0.42	3 240	4, 8, 10, 13, 16	409.41	5-24-35	3 578	7, 9, 20	384.23	382.92
95	1-26-34	0.98	3 043	4, 8, 13, 16	409.24	5-31-35	3 530	7, 9, 20	383.78	382.66
96	2- 2-34	0.86	3 049	4, 8, 10, 13, 16	409.54	6- 7-35	3 456	7, 9, 20	384.13	382.83
97	2- 9-34	0.14	2 862	4, 8, 10, 13, 16	409.57	6-14-35	3 738	7, 9, 20	385.30	383.57
98	2-16-34	0.02	2 813	4, 8, 10, 13, 16	409.51	6-21-35	3 755	7, 9, 20	384.67	383.13
99	2-23-34	1.71	3 086	4, 7, 8, 13, 16	409.00	6-28-35	3 089	7, 9, 20	384.85	383.22
100	3- 2-34	1.27	3 094	4,7,8,10,13,16	408.20	7- 5-35	2 959	7, 9, 20	383.80	382.64
101	3- 9-34	0.83	3 206	4,7,8,10,13,16	409.67	7-12-35	3 699	7, 9, 20	383.52	382.47
102	3-16-34	0.37	3 120	4,7,8,10,13,16	411.49	7-19-35	3 696	7, 9, 20, 11	383.38	382.38
103	3-23-34	0.00	3 165	4, 7, 10, 13, 16	411.59	7-26-35	3 397	7, 9, 20, 11	383.33	382.37
104	3-30-34	1.10	3 168	4, 7, 10, 13, 16	411.95	8- 2-35	3 475	7, 9, 20	383.07	382.18
105	4- 6-34	0.71	3 127	4, 7, 10, 13, 16	413.24	8- 9-35	3 424	7, 9, 20	382.82	382.00
106	4-13-34	1.76	2 862	4, 7, 10, 13, 16	413.55	8-16-35	3 261	7, 9, 20	382.68	381.92
107	4-20-34	1.78	2 646	4, 7, 10, 13	414.72	8-23-35	3 584	7, 9, 20, 12	382.50	381.87
108	4-27-34	0.31	2 567	4, 7, 10, 13	415.03	8-30-35	3 568	7, 9, 12	382.40	381.80
109	5—4-34	1.76	2 648	4, 7, 10, 13	414.83	9- 6-35	3 484	7, 9, 12	382.68	381.98
110	5-11-34	1.46	2 560	4, 7, 10, 13	415.16	9-13-35	3 404	7, 9, 12	383.38	382.38
111	5-18-34	0.42	2 781	4, 7, 10, 13	415.52	9-20-35	3 423	7, 9, 12	382.90	382.10
112	5-25-34	1.11	2 786	4, 7, 10, 13	415.35	9-27-35	2 968	7, 9, 12, 14	382.47	381.90
113	6- 1-34	0.04	2 780	4, 7, 10, 13	415.19	10- 4-35	3 582	7, 9, 12, 14	382.67	382.02
114	6- 8-34	0.93	2 816	4, 7, 10, 13	414.97	10-11-35	3 312	7, 9, 12, 14	382.53	381.97
115	6-15-34	1.89	2 762	4, 7, 10, 13	414.58	10-18-35	2 748	7, 9, 14	382.40	381.88
116	6-22-34	2.23	2 516	4, 7, 10, 13	414.70	10-25-35	2 010	7, 9, 14	382.52	381.95
117	6-29-34	0.38	2 841	4, 7, 10, 13	414.85	11- 1-35	3 307	7, 9, 14	382.65	382.02
118	7- 6-34	0.35	2 866	4, 7, 10, 13	414.10	....	....	....	......	......
119	7-13-34	0.56	2 742	4, 7, 10, 13	413.40	....	....	....	......	......
120	7-20-34	0.00	2 621	4, 7, 10, 13	412.77	....	....	....	......	......

* See Fig. 1. † See Fig. 2.

TABLE 2.—*Continued*

	(a) AT THE DIKE (*Continued*)				
Week No. (1)	Week ending (2)	Total rainfall, in inches (3)	Average pumping rate, in gallons per minute (4)	Location of pumping: Caisson*: (5)	Elevation of water surface, Morton Pond, in feet (6)
121	7-27-34	0.23	2 792	4, 7, 10, 13	412.32
122	8- 3-34	2.08	2 801	4, 7, 10, 13	411.93
123	8-10-34	0.00	2 762	4, 7, 10, 13	411.54
124	8-17-34	0.83	2 660	4, 7, 10, 13	411.06
125	8-24-34	1.93	2 127	7, 13	410.56
126	8-31-34	0.19	1 832	7, 13	410.58
127	9- 7-34	1.09	1 819	7, 13	410.05
128	9-14-34	3.12	1 478	7, 13	411.12
129	9-21-34	2.89	1 668	7, 13	411.81
130	9-28-34	0.54	1 362	7, 13	411.94
131	10- 5-34	2.26	1 165	7, 13	412.60
132	10-12-34	0.44	1 229	7, 13	412.90
133	10-19-34	0.08	1 148	7, 13	412.88
134	10-26-34	1.38	744	7, 13	413.03
135	11- 2-34	0.85	686	7, 13	......
136	11- 9-34	1.02	430	7	413.97
137	11-16-34	0.46	442	7	414.50
138	11-23-34	0.37	275	7	414.67
139	11-30-34	1.50	....	None	415.08
140	12- 7-34	1.06	....	None	415.94
141	12-14-34	0.07	....	None	416.31
142	12-21-34	0.72	....	None	416.51
143	12-28-34	0.91	....	None	417.00
144	1- 4-35	0.94	....	None	417.25
145	1-11-35	2.06	....	None	417.82
146	1-18-35	0.76	....	None	417.57
147	1-25-35	1.98	....	None	417.42
148	2- 1-35	0.11	....	None	417.35
149	2- 8-35	......	....	None	417.27
150	2-15-35	......	....	None	....

* See Fig. 1. † See Fig. 2.

The extent to which the ground-water was lowered, as read in the measuring wells and in the caissons, can be seen by inspection of Figs. 1 and 7 and of the data given in Table 2. In general, the ground-water was lowered more than 90 ft below the brook level over the deeper parts of the valley. The maximum air pressure used in sinking caissons other than Caisson 13 was 25 lb per sq in., only about 2% of the sinking requiring pressures in excess of 18 lb per sq in. (the legal limit in Massachusetts for 8-hr work). Altogether, 70% of the caisson sinking was accomplished in free air, and an additional 20% under pressures of less than 10 lb per sq in.

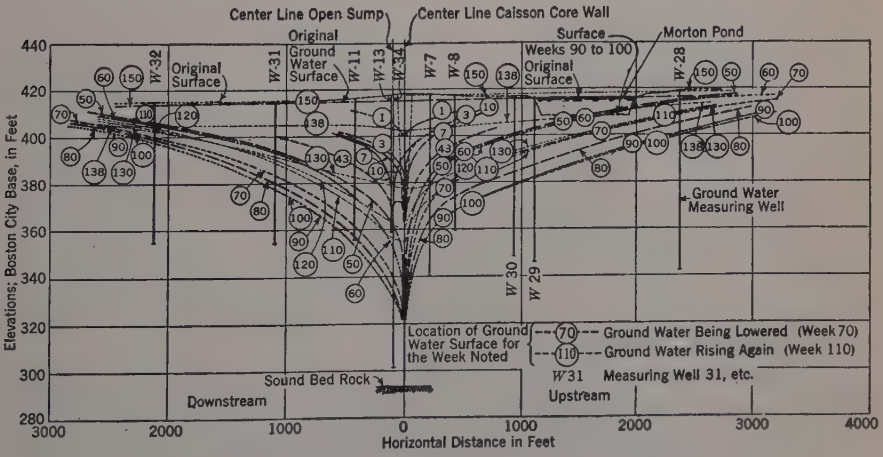

FIG. 7.—LONGITUDINAL SECTION OF BEAVER BROOK

As the core-wall of the concrete cassions neared completion, the pumping rate was decreased and the water level was allowed to rise again slowly while work progressed on the re-fill in the open trench excavated for caisson construction (see Figs. 1 and 7). In November, 1934, pumping was abandoned, and, in February, 1935, the water level was very close to its original position.

PERMEABILITY DETERMINATIONS FOR OVER-BURDEN FROM PUMPING OPERATIONS AT THE DIKE SITE

Several determinations of the permeability of the materials in the overburden were made from the data pertaining to pumping and ground-water lowering. The permeability was computed in various ways, but all determinations depended basically on the formula,

$$q = k\,p\,A\,S \qquad (1)$$

in which q = quantity of water flowing past any cross-sectional area, in cubic feet per day; k = the velocity of the water through the pores, in feet per day; and p = the porosity of the over-burden. Then $0.3267\,k\,p = K$, the permeability coefficient, in million gallons daily per acre for a slope of unity; A = area of cross-section, in square feet; and S = slope of the ground-water gradient (expressed as a decimal).

The value of q used in all determinations is based on the pumping rate. The total rate was composed of two general parts—infiltration and reduction of storage. The former consisted of surface infiltration of water from rainfall and from surface supplies, such as brooks, ponds, etc., and of sub-surface infiltration supplied by slow percolation from underground volumes previously unwatered of all that would flow out relatively quickly and easily. The latter consisted of the quantities taken from storage, which would immediately affect the readings of the ground-water measuring wells. (Estimates for pumping at the Main Dam and Dike indicated that about 90% of the water in the pores was taken quickly from storage by the pumping. Of the remainder, part would stay permanently in the pores by capillary action and part would be slowly underdrained.) The value of the flow, Q (in gallons per minute), at any location varies from the total quantity pumped at the station to practically zero at the radius of influence. The true value of the flow at the radius of influence depends upon the natural ground-water slope and the permeability (which is being determined). In some problems it may be necessary to make a preliminary assumption of this value to obtain a reasonable value of Q, but for this site the actual figure for the permeability finally obtained and for the slope would be less than 100 gal per min, so that the accuracy of the determinations will not be noticeably affected by the assumption.

The location of the radius of influence can be obtained by estimating visually, from the ground-water data, the distance from the pumping center at which the lowered ground-water surface will be tangent to the original ground-water surface. The value thus obtained, although approximate, is usable, because the exact location does not materially affect the permeability results. The values of Q at various locations between the pumping center and the radius of influence must be estimated as the first step in making a determination. Although, theoretically, an exact determination depends upon the exact variation in flow estimated, actually equivalent results will be obtained for any reasonable distribution of these differences of flow. The most reasonable distribution that was used in the Dike determinations was that Q was assumed to vary proportionately as the two factors—infiltration and storage reduction. The first varied as the surface drainage area, weighted for ponds and watercourses; and the second according to the rate at which the increment volumes of over-burden were being depleted of ground-water storage. Other arbitrary variations of Q also gave good results; for instance, Q can be varied from P to zero according to the distance from the pumping center, without seriously affecting the value of the determination. The empirical formula,

$$Q = P\left(1 - \frac{r^2}{r^2_i}\right) \text{.......................} (2)$$

seems to give a distribution of differences in flow very close to those obtained by the more careful and logical distribution first described. In Equation (2), P = the total pumping rate, in gallons per minute; r = a radius measured from the weighted location of pumping operations; and r_i = the radius that defines the influence of pumping and is the distance from the weighted pumping center to the limit where the pumped ground-water surface is tangent to the normal ground-water surface.

The symbol, A, denotes the area of the surface of the section through which the water flows, this section being curved to be taken perpendicular to the lines of flow. Near the pumping locations, this area has a bulbous shape due to the fact that the flow of water is not approximately horizontal near the intakes because: (1) There are several pumping intakes; and (2) these intakes are in the deeper parts of the valley. However, as the distance from the pumping center increases, these surfaces approximate very closely the surfaces of concentric vertical cylinders with axes passing through, and with radii taken from, the weighted center of pumping operations (herein termed the "pumping center"). The upper limits of these surfaces in determining the value of A are taken at the ground-water surface, as shown by readings in the measuring wells and the lower limits at the surface of the rock valley beneath, as determined from bore-hole investigations (see Figs. 1, 6, and 7). The use of these cylindrical areas does not introduce an approximation influencing the results, as the permeability determinations do not include volumes close to the pumping center.

The slope, S, is read directly from the ground-water curve used and is taken as a straight line between adjacent points on the curve. To avoid the introduction of errors (due to the use of the slope as a straight line, to the use of the average of end areas as the area of the volume considered, etc.), each determination was made by computing the permeability of sufficiently small increment volumes of the over-burden. Cylindrical cross-sections were

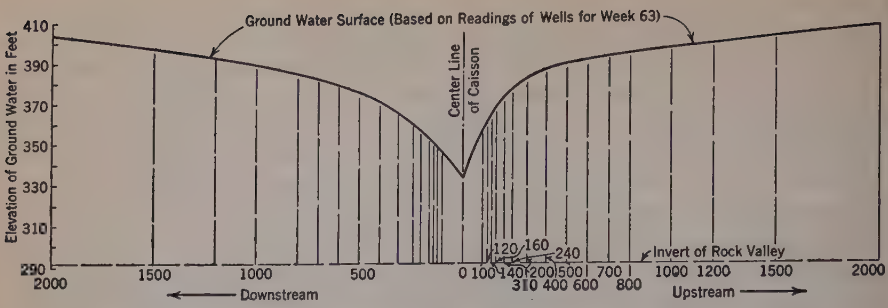

FIG. 8.—DEFINITION OF INCREMENTS FOR PERMEABILITY DETERMINATIONS

chosen at certain radii, as shown in Fig. 8, and each increment is the volume between cylindrical surfaces (see Table 3(*a*)). The effect of the size of the increment can be seen from the mathematical relation,

$$\frac{2\,\Delta}{2\,r + \Delta} = 2.3026\left(\log\frac{r_1 + \Delta}{r_1}\right) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which r_1 = the radius of the inside limit of the volume increment;

TABLE 3.—COMPUTATION OF PERMEABILITY BY "SLOPE QUANTITY" METHOD FOR WEEK 63 FOR BEAVER BROOK VALLEY (SEE FIG. 8)

(See Fig. 8) Distance from pumping center, in feet	(*a*) UP STREAM					(*b*) DOWN STREAM				
	Areas,* in thousands of square feet	Flow, Q,* in gallons per minute	Volume, V, in thousands of cubic feet	Slope, S	Permeability coefficient, K†	Areas,* in thousands of square feet	Flow, Q,* in gallons per minute	Volume, V, in thousands of cubic feet	Slope, S	Permeability coefficient, K†
0..........	....	....	....	......	......	....	....	....	......	......
100..........	20.0	1 910	....	......	......	16.7	1 446	....	......	......
	(22.5)	(1 908)	450	0.2000	26.7	(18.6)	(1 445.5)	372	0.1000	48.9
120..........	25.0	1 906	....	......	......	20.5	1 445	....	......	......
	(28.0)	(1 904)	560	0.1900	22.5	(23.0)	(1 444.5)	460	0.1000	39.5
140..........	31.0	1 902	....	......	......	25.5	1 444	....	......	......
	(34.25)	(1 899)	685	0.1750	19.9	(27.75)	(1 443.5)	555	0.1100	29.7
160..........	37.5	1 897	....	......	......	30.0	1 443	....	......	......
	(43.25)	(1 893)	1 730	0.1425	19.3	(33.8)	(1 442)	1 352	0.1050	25.5
200..........	49.0	1 889	....	......	......	37.6	1 441	....	......	......
	(56.0)	(1 881)	2 240	0.1150	18.3	(41.8)	(1 438)	1 672	0.0925	23.3
240..........	63.0	1 873	....	......	......	46.0	1 436	....	......	......
	(74.5)	(1 859)	5 215	0.0743	20.6	(56.0)	(1 432)	3 290	0.0857	18.8
310..........	86.0	1 845	....	......	......	66.0	1 428	....	......	......
	(99.5)	(1 827)	8 950	0.0444	26.0	(79.5)	(1 423)	7 160	0.0588	19.2
400..........	113.0	1 809	....	......	......	93.0	1 418	....	......	......
	(123.5)	(1 777)	12 350	0.0250	36.2	(102.5)	(1 411)	10 250	0.0430	20.1
500..........	134.0	1 746	....	......	......	112.0	1 405	....	......	......
	(138.0)	(1 714)	13 800	0.0180	43.5	(116.0)	(1 398)	11 600	0.0360	21.0
600..........	142.0	1 683	....	......	......	120.0	1 392	....	......	......
	(145.0)	(1 645)	14 500	0.0190	37.5	(126.0)	(1 379)	12 000	0.0280	24.5
700..........	148.0	1 608	....	......	......	132.0	1 366	....	......	......
	(151.0)	(1 571)	15 100	0.0150	43.5	(132.5)	(1 354)	13 250	0.0240	26.9
800..........	154.0	1 534	....	......	......	133.0	1 341	....	......	......
	(159.0)	(1 464)	31 800	0.0145	40.0	(134.5)	(1 289)	26 900	0.0210	28.6
1 000..........	164.0	1 394	....	......	......	136.0	1 238	....	......	......
	(169.5)	(1 312)	33 900	0.0120	40.5	(141.5)	(1 164)	28 300	0.0185	28.0
1 200..........	175.0	1 229	....	......	......	147.0	1 090	....	......	......
	(183.0)	(1 112)	54 900	0.0110	34.8	(152.5)	(983)	45 700	0.0166	24.3
1 500..........	191.0	994	....	......	......	158.0	876.5	....	......	......
	(204.5)	(815)	102 200	0.0114	22.0	(169.0)	(704)	100 500	0.0119	21.9
2 000..........	218.0	636	....	......	......	180.0	532	....	......	......

* Values in parentheses are averages. † In millions of gallons per acre per day at 50° F for a slope of 1.

$\Delta = r_2 - r_1$; and r_2 is the radius of the outside limit of the increment. By substituting the values of r and Δ considered, the right side of Equation (3) represents the undistorted value. Any values of Δ that will not seriously distort the left-hand side beyond the limits of accuracy required can be used. Estimates of the locations of the limits of such increments to eliminate the errors show that, if the slope can reasonably be taken as a straight line between those limits, good results can be obtained by using radii of 50, 100, 150, 200, 300, 400, 500, 600, 800, 1 000, and 1 300 from the pumping center.

The influence of materials beyond about 2 000 ft from the pumping center upon the permeability determination is not considered because: (1) As the ground-water curve becomes flatter, the accurate determination of the permeability for that region becomes more difficult, slight differences in elevations of readings making material differences in results; (2) such materials are out of range of the immediate site; (3) the ground-water level near the radius of influence is more sensitive to fluctuations due to rainfall and seasonal influences than those due to the pumping; and (4) for the foregoing reasons, ground-water elevations were not read at this site.

Furthermore, volumes close to the caissons were not considered for four reasons: (1) There were no ground-water measuring wells near the caissons because of the open-cut trench and, therefore, the elevation of the ground-water surface at that point was not determined accurately; (2) cylindrical cross-sectional areas cannot be used accurately as the areas of flow; (3) the pumping operations and the compressed-air work during the sinking of the caissons so distributed the permeability of the materials immediately surrounding the caissons that those portions of the over-burden should not be considered as representative in obtaining the effective average; and (4) these portions represent relatively small volumes, and as the permeabilities of increments are weighted according to their volumes, the resulting determination would not be seriously influenced by the values obtained for these volumes in any event.

The shape of the rock valley at this site naturally divides the ground-water flow toward the pumps into two parts, that from up stream and that from down stream. A study of the rainfall and run-off conditions at the site and of the storage depletions up stream and down stream from the pumping center indicated that on the average about 57% of the total pumpage comes from the valley over-burden up stream and the remaining 43% from the valley over-burden down stream. Whether or not this indication is true, the result for the average effective permeability of the valley as a whole is affected but slightly by it, due to the compensating effect of the averaging of the two values to obtain the result, as can be seen from an examination of Table 4.

Using the factors, q, A, and S, the permeabilities of increment volumes of the over-burden can be obtained for any ground-water surface conditions and the corresponding pumping rate. By combining the permeabilities of the increments thus obtained, and weighting each increment according to its volume, the effective average permeability of the whole material is obtained, this method being called herein the "slope quantity" method.

TABLE 4.—COEFFICIENTS OF PERMEABILITY

Determination for:	Assumed Percentages of Total Pumping		K=Coefficient of Permeability		
	From up-stream volumes	From down-stream volumes	Up stream	Down stream	Weighted average
Week 63	70	30	37.7	16.7	27.9
	57	43	31.9	24.0	28.2
	50	50	29.0	27.9	28.0
Week 91	77	23	45.1	16.2	32.0
	70	30	43.3	17.8	31.8
	57	43	35.2	25.5	30.9
	50	50	31.0	29.7	30.4

Fig. 8, with Table 3, shows the details of such a determination for Week 63 of the pumping. The best results can be obtained by choosing a week among those for which conditions (such as the pumping rates, depletions of storage, and uniform surface infiltrations due to normal rainfalls and run-offs) have been running constant or varying uniformly. The measuring wells register sensitively the changes in ground-water surface. For a period when the rainfall is uniformly distributed, the quantity of infiltration is probably constant. If the pumping plant operates at a constant rate, the reduction in storage is a constant volume from week to week. For such periods accurate estimates of the quantity of water furnished by storage depletion and by surface infiltration can be made. Such estimates can be used in determining accurately, the quantity of water flowing toward the pumps at various vertical cross-sectional areas (see Fig. 8 and Table 3). As the quantity flowing between any two vertical cross-sections and the loss of head are then known, the permeability of any corresponding increment volume can be determined accurately. Weeks 63 and 91 are of that nature and, therefore, are used. Table 5, introduced subsequently, gives certain of the better determinations made as described herein. That the value of K is constantly increasing as the water level recedes is accounted for by the fact that the upper portions of the glacial valley over-burden consist of finer sands containing streaks of rock flour and the lower portions contain the coarser lenses. Thus, as the ground-water level is lowered beneath the finer portions, the resulting average permeability of portions under the ground-water surface is greater.

It is interesting to note that in all determinations practically the same result can be obtained by using Q = three-fourths of the up-stream pumpage, P_u, as the flow for the up-stream increments, and three-fourths of the down-stream pumpage, and P_d, as the flow for the down-stream increments. This is probably due to the facts that: (*a*) The K-values of the increments do not vary to such degrees as to upset the results; (*b*) the manner for distributing the infiltration is approximately a formula the equation of which is of the second degree; and (*c*) the storage depletion increments vary roughly as the square of the distance from the caisson (the rock valley being not far from a constant cross-section). It can be inferred, therefore, that reasonable approximations can be made for like conditions by adopting a constant value of Q instead of the variable one used in the more exact determinations.

The results of the work done by this increment permeability ("slope-quantity") method can be checked by a second cut-and-try method described subsequently as the "graphical method". In the former an estimate of the portions of the total pumpage coming from up stream and down stream is necessary. Although the accuracy of such an estimate does not seem to affect the average effective permeability of the over-burden as a whole, it will affect the relative values obtained for the up-stream and down-stream permeabilities. In the latter an assumption is made that the average effective permeabilities of the up-stream and down-stream portions are equal, which may or may not be true for any location considered. However, the results by either method are confirmatory, showing that the manipulation of these considerations within the reasonable limits indicated will not seriously affect the workability of the methods.

Another more approximate method of obtaining the permeability from ground-water pumping data is by an adaptation of the formula[9] used generally for determining available ground-water yield from wells:

$$y^2 = \frac{q}{k\,p} \log_e \frac{x}{r} + h^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which x and y are co-ordinate locations of any point on the ground-water curve in reference to co-ordinate axes, one axis being made to pass vertically through the pumping center and the other horizontally at the average elevation of rock. Furthermore, h = the y-co-ordinate at the pumping well; p = porosity; and k = the velocity of the ground-water through the pores, in feet per day, for a slope of 1 on 1 at 50° F.

Equation (4) can be used only in its present form for an accurate determination in a case where the pumping curve has reached a stable position, q being a constant flow from beyond the radius of influence to the pumps with no depletion in storage and with no infiltration within the radius of influence, and for a condition which considers a level rock floor under the over-burden extending 360° in plan around the pumps and to the radius of influence. (Even then the formula is for well-points and assumes horizontal flow near the pumping intakes.) Approximations in the field, of such theoretical conditions, for any dam location, or for any reasonable pumping or unwatering program, are extremely unlikely.

Approximate and useful results, however, might be obtained for some cases by adapting the principles of the formula, as follows: The average of the up-stream and down-stream ground-water curves can be obtained, as illustrated in Fig. 9. An equivalent rock floor, which will extend 360° around the pumping center, can be estimated by summing the areas up stream and down stream for various radii and dividing by $2\,\pi\,r$; and the horizontal plane which best averages the location of this rock floor so determined can be estimated roughly by eye, as shown in Fig. 9. A constant Q-value of three-fourths P can be used. Then, by substituting values of x and y (of any two points on the average ground-water curve in relation to the base fixed) twice

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[9] For derivation see "Public Water Supplies", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and H. L. Russell, M. Am. Soc. C. E., Second Edition, p. 279.

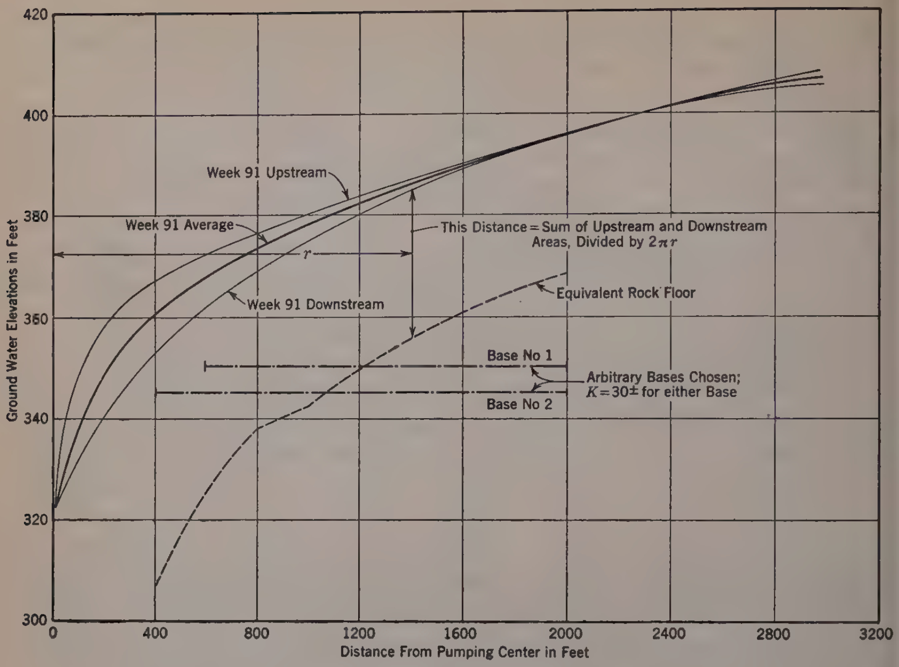

FIG. 9.—GROUND-WATER ELEVATIONS AT THE DIKE

in Equation (4), two equations, with values, $K p$ and h, unknown, can be obtained. These equations can be solved simultaneously for $K p$ by subtraction. A weighted mean of a number of such determinations would give the average effective permeability. In Equation (4) h is usually a difficult factor to obtain accurately, especially in cases of pumping from many locations, but where it can be obtained, permeability determinations can be made for as many points on the average curve as seem desirable without the necessity of solving simultaneous equations.

PUMPING OF GROUND-WATER AT THE SITE OF THE MAIN DAM

The first of the exploratory caissons (Caisson 20, see Fig. 2) was sunk to a depth of 75 ft below the river level, at a location which was not in the deeper part of the rock valley. This caisson was equipped with intakes for pumping and two pumps with capacities of 300 gal per min were installed. Ground-water was pumped for two months in 1932 at an average rate of about 650 gal per min and the ground-water surface was lowered only about 7 ft in the vicinity of this caisson, because of the lack in capacity of the pumping intakes. Pumping was then abandoned at this site for about a year, and the ground-water surface returned to its original position. The capacity of the pumping intakes was increased in July, 1933, mainly by the forcing of well-points out into the surrounding materials through holes drilled in the sides of the caisson. Two additional pumps (capacity, 800 gal per min) were installed, and pumping was started at an increased rate. An average of about 2 100 gal per min was pumped over the period of the next eleven months, the water level being lowered about 30 ft near the caisson. The pumping rates and the progressive lowering of the ground-water may be seen in Fig. 2 and Table 2(*b*).

The Swift River, which flows past the site, was diverted in June, 1933, through a stream-control conduit in the hillside at the westerly end, by two low embankments built across the old river bed, one at about 800 ft up stream from the center line of the Main Dam and the other about 1 300 ft down stream. The run-off from the intermediate drainage area was allowed to flow into the part of the former river bed between these embankments, and the pumpage was discharged at this point also, an opening being maintained through the down-stream embankment. In December, 1933, a 24-in. discharge line was built to the river up stream from the low embankment, and the opening through the down-stream embankment was closed. The subsequent pumping operations quickly dried up the old river location between embankments.

The practicable limit of lowering by pumping from Caisson 20 was reached in the spring of 1934, because of the elevations of the pumping intakes.

Two additional exploratory caissons (Caissons 7 and 9, Fig. 2) were then sunk in the summer and fall of 1934 in the deep part of the valley to investigate further the over-burden and to demonstrate the feasibility of further lowering the ground-water. All these caissons later became parts of the finished concrete core-wall, although they were used temporarily for experimental purposes. During the sinking of these caissons pumps were operated alternately in them, materials being excavated under air from one, while the ground-water level was being lowered in the other with pumps installed in the working chamber. During this period, pumping continued from Caisson 20 at a diminishing rate, averaging about 1 600 gal per min at the start and decreasing gradually, until at the completion of these caissons, five months later, it was about 1 200 gal per min. In addition, pumping from Caisson 9, when used as a pumping plant, averaged about 1 200 gal per min, and from Caisson 7 about 2 500 to 3 000 gal per min. The effect of this pumping upon the ground-water level is shown in Fig. 2 and Table 2(*b*).

In this manner, these two caissons were sunk to ledge approximately 120 ft below the river level and about 100 ft below the ground-water level as lowered at the beginning of sinking, the maximum air pressure used in Caisson 7 being 37.5 lb per sq in. and in Caisson 9, 34 lb per sq in. When these caissons had been completed, pumping was continued from them and from Caisson 20. The average total rate at the start was 5 000 gal per min. (3 100 gal per min from Caisson 7, 1 000 gal per min from Caisson 9, and 900 gal per min from Caisson 20). Two months later, in February, 1935, the rate had decreased to 3 400 gal per min. (2 200 gal per min from Caisson 7, 1 000 gal per min from Caisson 9, and 200 gal per min from Caisson 20), and the ground-water level had been lowered about 75 ft in the vicinity of Caissons 7 and 9, as shown in the curves of Fig. 2.

Pumping was continued at approximately this rate for several months, maintaining the ground-water in its lowered position. The work of constructing the caisson core-wall began in May, 1935, and was practically completed by December, 1935, during which period the contractor pumped an average total of about 3 400 gal per min from Caissons 7 and 9 and other caissons, and such pumping did not materially lower the ground-water gradients, except that in the center of the rock valley, as shown in Fig. 2, further local lowering was secured near Caissons 12 and 14. The maximum air pressure used in sinking caissons other than Caissons 7, 9, and 20, was a little in excess of 25 lb per sq in., although nearly all caissons were sunk under pressures of less than 18 lb per sq in. About 82% of the sinking was accomplished in free air, Caissons 1, 2, and 32 to 40, inclusive, being entirely sunk and sealed without the use of air under pressure.

PERMEABILITY DETERMINATIONS FROM PUMPING AT THE MAIN DAM

Determinations of the permeability of the over-burden at the Main Dam can be made from the curves plotted showing the ground-water surfaces at various intervals during the pumping. The determinations can be made in addition to the "slope quantity" method described previously by "the graphical" (a cut-and-try) method. The procedure may be demonstrated by reference to Fig. 10, in which Curve *A* is plotted from measuring-well readings; Curves *B*, *C*, and *D*, for the values of *K* indicated, represent the total pumpage divided by 54% from up stream and 46% from down stream; and Curve *E*, for $K = 30$, represents the total pumpage divided by 60% from up stream and 40% from down stream. Preparation for a determination by this method consists of plotting in plan over the rock contours of the valley floor, the con-

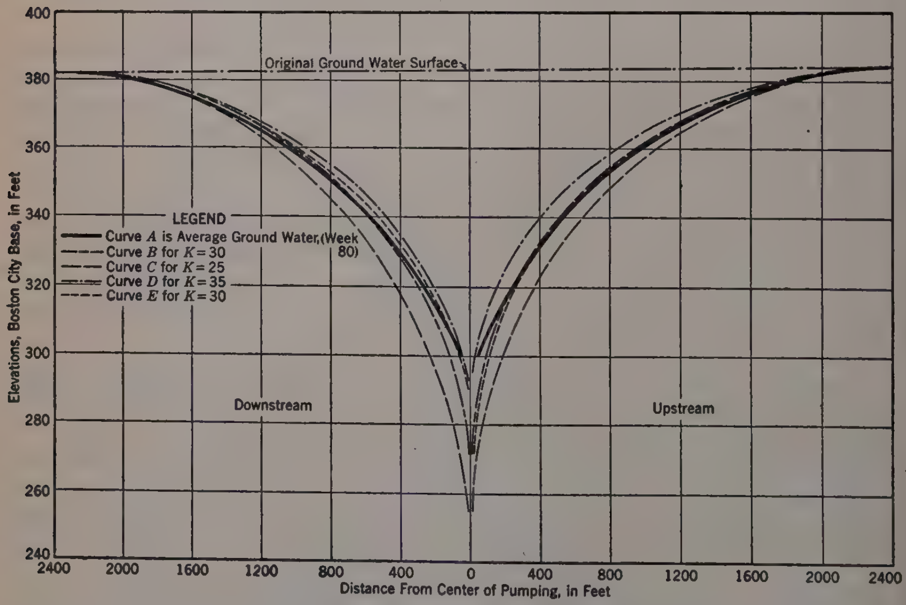

FIG. 10.—PUMPING AT THE SITE OF THE DAM

tours of the ground-water level from the readings of the ground-water measuring wells (see Fig. 11). A pumping center is chosen as in the other methods, in this case a point about 140 ft northeast of Caisson 7 on the center line of the Main Dam being taken. With this center, radii of 150, 200, 300, 400, 500, 600, 800, 1 000, and 1 300 ft are used, these radii being taken so as to give increments small enough to eliminate errors as described previously. The cylindrical surface, cross-sectional areas between the surface of the underlying rock and the surface of the ground-water contours, are then estimated for sections located at the radii taken. The quantity, Q, flowing between cylindrical surfaces at various distances from the pumping center is assumed to vary in this case according to Equation (2), varying the total quantity flowing from practically zero at the radius of influence to the total pumpage at the pumping center.

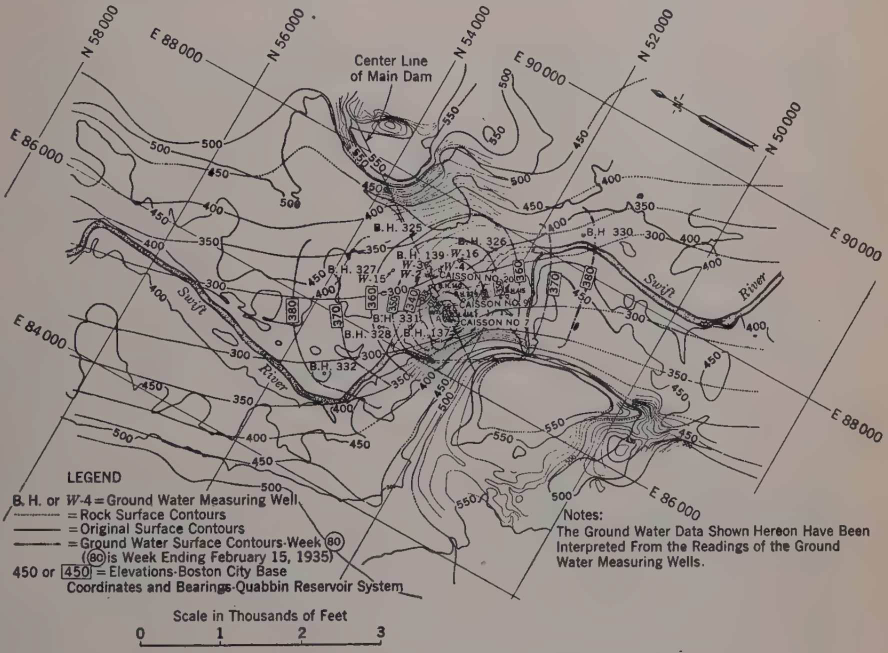
N 58 000
E 88 000
N 56 000
Center Line of Main Dam
N 54 000
E 90 000
N 52 000
N 50 000
E 86 000
E 84 000
E 90 000
E 88 000
E 86 000
Swift River
Swift River
B. H. 325
B. H. 326
B. H. 327
W-15
B. H. 139 W-16
W-3
W-4
B.H. 330
CAISSON NO. 20
CAISSON NO. 9
CAISSON NO 7
B.H. 331
B.H. 328
B.H. 137
B.H. 332
360
350
340
380
370
500
450
400
350
300
550
LEGEND
B. H. or W-4 = Ground Water Measuring Well
= Rock Surface Contours
= Original Surface Contours
= Ground Water Surface Contours-Week (80)
((80) is Week Ending February 15, 1935)
450 or [450] = Elevations-Boston City Base
Coordinates and Bearings-Quabbin Reservoir System
Notes:
The Ground Water Data Shown Hereon Have Been Interpreted From the Readings of the Ground Water Measuring Wells.
Scale in Thousands of Feet
0
1
2
3

FIG. 11.—TOPOGRAPHY AT THE SITE OF THE MAIN DAM

The cut-and-try "graphical" method consists of the following steps:

(1) Divide the pumpage into up-stream and down-stream portions, assuming a certain percentage of the total from up stream and the remainder from down stream.

(2) Assume that the average effective permeability of the down-stream and up-stream over-burdens are the same.

(3) Assume a value of K which seems reasonable.

(4) Using these assumed factors, and the computed cross-sectional areas obtained from the ground-water surface for which the determination is made, theoretical ground-water curves up stream and down stream can be plotted

in a vertical plane running longitudinally to the valley, starting at the radii of influence (as determined from the pumping curve data) and plotting in toward the pumping center (see Fig. 10). These curves are found by computing the lost head between each set of cylindrical surfaces by use of the formula:

$$H = \frac{62.88\, Q\, (r_2 - r_1)}{\frac{a_2 + a_1}{2}\, K} \quad \text{.....................} (5)$$

(5) The curves of Step (4) are then superimposed on the curve of the ground-water surface formed by the intersection of the vertical longitudinal plane with the ground-water surface contours plotted (see Fig. 10).

(6) If all the points so plotted come above the observed ground-water curves, the permeability assumed is too large; if they are below, the permeability is too small. If the up-stream points are plotted below, and the down-stream points above, the division of the percentage of flow is incorrectly assumed.

(7) By repeating assumptions, a closer and closer solution for the permeability can be obtained, as well as a better idea of the division of flow coming from up stream and down stream. The nearness of the solution to the correct average effective permeability will be demonstrated graphically by the manner in which the computed curves coincide with the observed ground-water curves.

In all cases where determinations were made by the two methods, the results obtained with the "graphical" method check very nicely those obtained by the analytical "slope quantity" method (see Table 5).

TABLE 5.—PERMEABILITY RESULTS, IN MILLION GALLONS DAILY PER ACRE, FOR A SLOPE OF 1 AT 50° F.

Site	PART I, BORE-HOLE CLASSIFICATION		PART II, PUMPING DATA				
				Slope Quantity Method			
	From sieve analyses	From tested permeabilities	Week No.	Approximate $\left(\frac{3}{4}Q\right)$	More Exact (Q Computed)	Graphical Method	Well Formula
					(See Fig. 8 and Table 3)	(See Fig. 10)	(See Fig. 9)
Dike...........	19	21	5	..	..	..	16
			36	..	..	21	..
			63	24	28	..	..
			91	28	32	30	30
			133	23	..	..	..
Main Dam.......	24	28	30	29	..	33	..
			62	40	..	44	..
			80	29	..	31	..

COMPARISON OF PERMEABILITY DETERMINATIONS

An inspection of Table 5 shows that the coefficient of permeability at 50° F of the entire over-burden of the Dike is about 20 mgd per acre and of the lower parts of the over-burden about 30 mgd per acre; and that it is about

30 to 35 mgd per acre for the entire over-burden of the Main Dam. The result of all methods, whether obtained by the use of the bore-hole samples and laboratory work, or by the use of the pumping and ground-water data, substantiates these figures. Considering that a permeability determination within the limits of accuracy of 50% to 200% is extremely useful and sufficiently close for most design purposes, the results in Table 5 demonstrate the adequacy of the different methods for making determinations well within desired limits.

The accuracy of determinations made from the classification of bore-hole samples depends to a great extent upon the manner in which bore-holes are located and the samples taken and collected, upon the skill with which the field and laboratory work is conducted, and upon the judgment as to the manner in which the various classes of material exist in the over-burden. From experiences with the work described herein, the writer is confident that a similar application of the "bore-hole sample classification" method will give valuable results at other locations if reasonable care is exercised in the application.

The use of ground-water pumping to secure a permeability determination is much simpler, provided there is sufficient information regarding the pumping rates, the location of the ground-water surface, and the location of the rock valley. The ground-water surface is extremely sensitive and measures accurately the loss of head between points in the valley. This registration of lost head makes a determination by any of the foregoing methods a comparatively simple and accurate one, because very small differences in permeability will give large differences in readings of the ground-water surface in the measuring wells. Unreasonable determinations can easily be proved unworthy because the loss of head for such conditions will not agree with those observed. These pumping methods can be used elsewhere where the rock and ground-water surfaces are known, to determine the effective permeability of foundations below the ground-water surface.

Conclusions

There seems to be a scarcity of data concerning the permeable characteristics of soils and there are few, if any, definite methods in common use for making satisfactory determinations of the permeability of an over-burden. In many cases permeable qualities of foundations have been a matter of extremely rough estimate, of guess, or merely of conjecture. The aforementioned methods, although not entirely satisfactory, will be found useful in that connection for conditions similar to the ones described, and such methods or adaptations of them may prove helpful elsewhere where conditions are dissimilar. Although the results of determination by such methods may not stand too rigid tests for complete exactness, they will prove their value if only they can be used to serve as satisfactory guides in formulating the opinions or judgment of the engineer confronted by the problem of how large is the permeability of the foundations under consideration.

Acknowledgments

Most of the information used for the preparation of this paper was obtained from data secured by the Engineering Department of the Metropolitan District Water Supply Commission of Massachusetts, of which Frank E. Winsor, M. Am. Soc. C. E., is Chief Engineer and Karl R. Kennison, M. Am. Soc. C. E., is Assistant Chief Engineer. The writer is indebted to many members of this organization for aid in preparing and compiling much of the information and for constructive suggestions on, and pertinent criticisms of, the substance of the paper itself.

GEOLOGIC FEATURES, QUABBIN AQUEDUCT

BY FRANK E. FAHLQUIST[10], ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The geological studies which were of importance in constructing the Quabbin Aqueduct, of the Metropolitan Water Supply of Boston, Mass., are presented in this paper. The general geology of the area, including that of the surface and that of the underground, is discussed in relation to the several practical problems encountered during the location studies. Data pertaining to the behavior of the different rocks during construction and the cost and progress of excavation through them, are also included.

INTRODUCTION

Construction of the Quabbin Aqueduct is one of several steps in extending the water supply system of the Boston Metropolitan District beyond the Wachusett water-shed in Central Massachusetts (see Fig. 12). In the past water has been obtained from several minor sources and the major supply of Wachusett Reservoir, formed by impounding the flow of the Nashua River, at Clinton, Mass. In extending this system westward to the Swift River Valley additional storage and supply are made available. Completion of the easterly portion of the aqueduct, extending between the Wachusett Reservoir and the Ware River, near Coldbrook, Mass., marked the first step in this project. Construction of this section was sufficiently advanced by March, 1931, to permit diversion of the water of the Ware River to the Wachusett Reservoir. Construction of the western portion of the aqueduct, extending from the terminus of the East Tunnel section to the portal and intake at the Quabbin Reservoir, was completed during the summer of 1935. The result

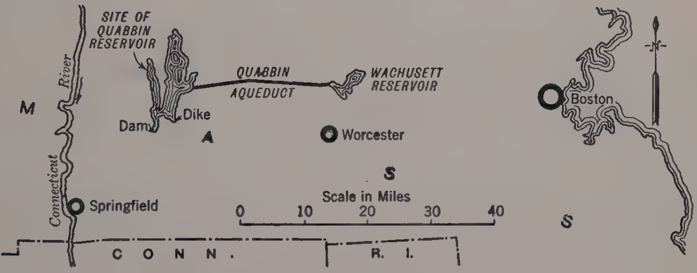

FIG. 12.—LOCATION OF QUABBIN AQUEDUCT AND SITE OF QUABBIN RESERVOIR.

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[10] Senior Civ. Engr., Met. Dist. Water Supply Comm., Enfield, Mass.

is a continuous aqueduct, 24.6 miles long, connecting the Quabbin Reservoir with the Wachusett Reservoir.[11]

GEOLOGIC SETTING

The tunnel crosses a part of the central upland of Massachusetts known as the Worcester County Plateau, a large part of which stands between 1 000 and 1 200 ft above sea level. Geologists[12] have noted an unusual uniformity in the horizon line of the district, and have recognized in the general correspondence of the summits of the parallel ridges the former existence of an ancient and nearly uniform surface, or peneplain, sloping gently toward the southeast.

In general, the hills and valleys are oriented in a north-south direction, which is closely parallel to the folds of the rock formations of the area. There are exceptions, however, which are sufficiently numerous to indicate that dissection or erosion of the ancient surface, or peneplain, was not controlled entirely by the distribution and structure of the underlying rock formations. An explanation for this irregularity has been advanced by W. O. Crosby[13], who concluded that, at the time of its fullest development, the peneplain was covered by a thick mantle of sediment, upon which a drainage pattern was developed without much regard for the structure and distribution of the underlying hard and soft rocks. At this stage the stream gradients were steepened and the rivers rejuvenated as the result of crustal disturbances. As a result the rivers became deeply entrenched in the thick mantle of unconsolidated deposits. By the time these deposits were removed, the streams had been superimposed on the older crystalline rock floor. The pattern of this ancient drainage system can be traced and reconstructed, in part, except in those areas where the valleys have been completely blocked, and where features of the bed-rock topography have been buried too deeply by glacial deposits.

As the glacier advanced during the succeeding epoch known as the Glacial Period, several of these valleys were slightly deepened and widened, and the topography was modified somewhat by ice erosion. As it retreated from the area large quantities of rock material, which had been gathered by, and frozen into, the ice, were deposited. In general, this glacial drift material was deposited to rather shallow depths in the higher elevations, but accumulated to considerable thickness within the valleys.

PRE-GLACIAL DRAINAGE COURSES

Many of the streams of this section are too small in size to have excavated the broad and deep depressions which they now occupy. There are also numerous notches or gaps, situated across divides which are now unoccupied by streams. These features, which are duplicated many times throughout New England and other sections of the United States, have long been recognized by geologists as being related to the pre-historic or pre-glacial river systems developed before the Glacial Period.

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[11] Accounts by Frank E. Winsor, M. Am. Soc. C. E., and Karl R. Kennison, M. Am. Soc. C. E., have appeared recently in engineering literature, as follows: "Boston's New Metropolitan Water Supply", by Frank E. Winsor, *Civil Engineering*, Vol. 4, No. 6, June, 1934; "Ware River Intake Shaft and Diversion Works", by Karl R. Kennison, *Civil Engineering*, Vol. 4, No. 8, August, 1934; and "Boston Metropolitan Water Supply Extension", by Karl R. Kennison, *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2, June, 1934.

[12] "Physical Features of Central Massachusetts", by William C. Alden, *Bulletin 760*, U. S. Geological Survey, 1924, pp. 13–105.

[13] "Geological History of the Nashua Valley During the Tertiary and Quaternary Periods", by W. O. Crosby, *Technology Quarterly*, Vol. 12, 1899, pp. 288–324.

Reconstructing the pre-glacial drainage pattern was important in the tunnel location studies in indicating areas where the underlying rock floor might be lower in elevation than was apparent from a study of surface geological conditions. The limits of many of the pre-glacial valleys are obscured by extensive glacial drift deposits of uncertain thickness. Because of the uncertainty in the position of the rock floor, such areas were recognized as being more important than others, where geologic conditions were more easily interpreted. These preliminary investigations outlined the critical areas throughout which more detailed investigations should be made by borings. Certain areas thus outlined were eliminated from serious consideration at an early stage of the investigations, as the gaps or notches situated within some of the valleys occur at such high elevations that there was little possibility of a gorge extending to the tunnel grade. The most important pre-glacial drainage courses are shown in Fig. 13, together with the boundaries of the two principal types of glacial drift deposits.

Glacial Drift Deposits

The greater part of the area traversed by the tunnel is covered by a glacial soil known to the geologist as a ground moraine and sometimes to the engineer as hardpan. This deposit occurs as a heterogeneous mixture of rock flour, sand, gravel, and boulders. In general, it is found in areas of higher elevation, and is usually deposited to relatively shallow depths. Recognition of the fact that, in general, the ground moraine is a shallow deposit, was found helpful in estimating bed-rock configuration, and also in delineating the less critical areas where explorations by borings were not necessary.

When the glacier was receding, many of the valleys were blocked periodically by glacial drift and ice. Water derived from the melting ice was thus ponded, forming temporary glacial lakes which received large quantities of well assorted deposits. This phase of the glacial history may be recognized in the character and mode of occurrence of the deposits, which consist largely of stratified fine sand and banded clay or rock flour. The surface topography within these lake areas is generally flat. One of the most notable examples of such a temporary lake was the glacial Lake Nashua, cited by Crosby[13], which covered a part of the Nashua River drainage area. The present Wachusett Reservoir occupies a part of the bed of this lake.

Valleys with outlets that were kept open throughout the period of glacial recession contain deposits of a modified character, the irregular structure and coarseness of which indicate an origin other than that in the quiet waters of a glacial lake. The lower sides and bottoms of these valleys contain irregularities in the general surface topography in the form of ridges and terraces composed chiefly of sand and gravel. These deposits indicate deposition by glacial stream action. Some were deposited in front of the retreating glacier

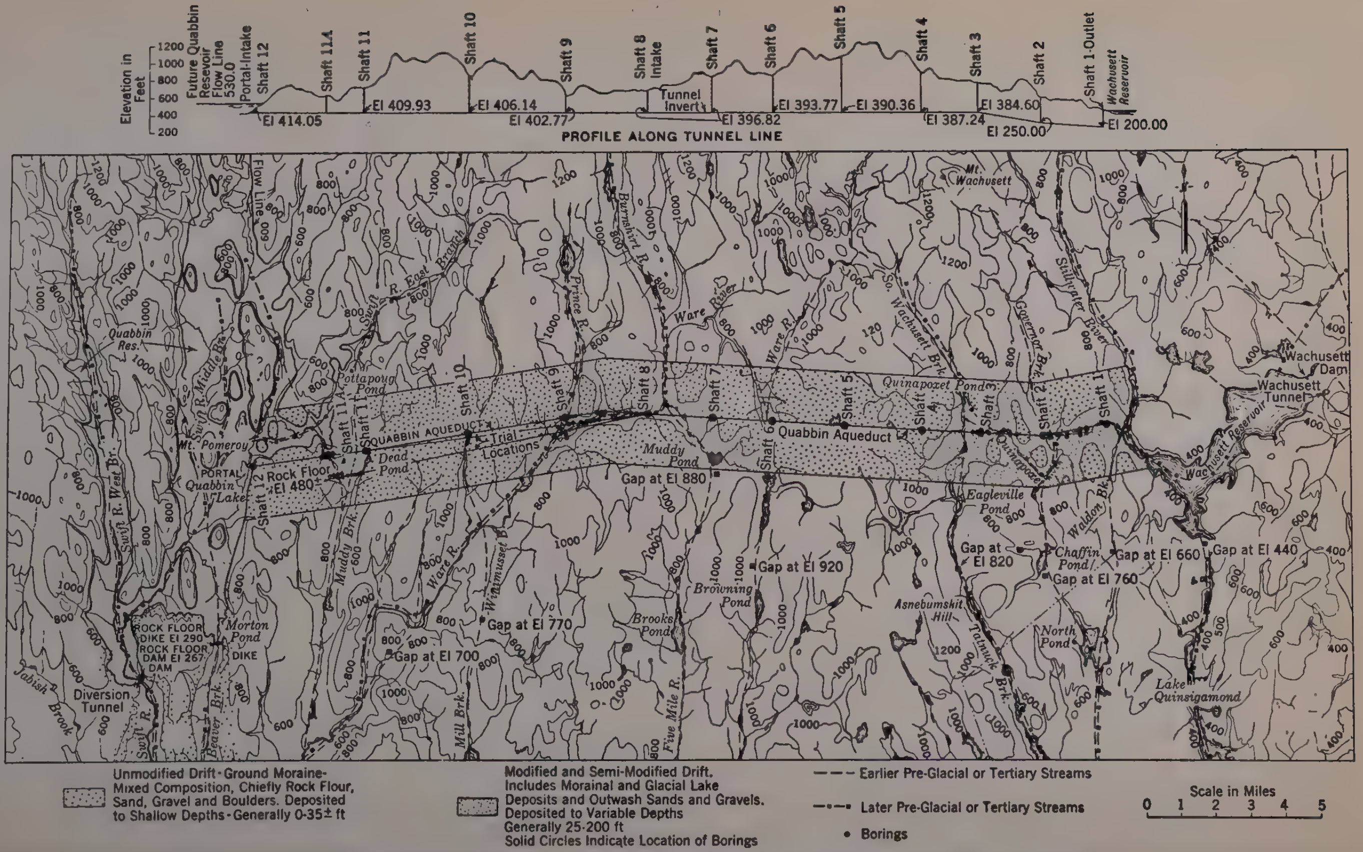

FIG. 13.—DISTRIBUTION OF GLACIAL DRIFT AND PRE-GLACIAL DRAINAGE COURSES.

as a sheet or outwash, some beneath the ice in sub-glacial river channels, and others on top of the ice. As a whole, they form a curious assemblage of flat terrace-like areas, and steep and gradually sloped knolls and ridges of varying magnitude.

There are also other deposits of a semi-modified character the surface character and form of which indicate an origin near the front of the glacier. Alden[12] has recognized these deposits as recessional moraines, has traced their approximate boundary lines, and has differentiated the several stages of glacial retreat which contributed toward their development. In surficial configuration these deposits form knolls and ridges of irregularly stratified sand and gravel. Bowl-like depressions, many of which contain small ponds and bogs, are interspersed throughout the morainal area. The difference in relief of these humps and hollows range from 5 ft to not more than 100 ft.

Differentiating between the foregoing types of glacial drift, which occur within the tunnel area, was helpful in choosing the areas throughout which knowledge of the geologic conditions was more uncertain. These modified and semi-modified drift deposits, compared to the unmodified ground moraine, occur in much greater thickness. Their occurrence at several points along the tunnel line introduced uncertainties as to the location and quality of the underlying rock, which, finally, could be determined only by borings.

Tunnel Locations

Geological Information Required.—The particular kind of geological information necessary for solving the engineering requirements of the Quabbin Aqueduct may be illustrated best by presenting certain questions raised from time to time in developing the project. Perhaps the following questions will serve the purpose of indicating the nature and range of those presented during the investigation:

Of several possible tunnel locations, is there any geological reason for choosing one line rather than another? Which parts of the location require exploratory investigations beyond what can be determined by field inspection of the surface topography and rock ledges? After preliminary borings have been made, can one tell whether the purpose in each case has been fully met or are additional explorations necessary?

What is the maximum depth of buried pre-glacial stream channels crossing the tunnel line as bearing on the grade of the tunnel? Would a shift of the line avoid any uncertainty as to buried channels and thus end the question of further explorations? Can shafts be located at comparatively low spots and at the same time be reasonably distributed, and yet avoid locations where the cover is heavy or where troublesome conditions may occur?

How many different rock formations will be encountered? what is their individual character, structural relation, and sequence, and what are the practical differences as affecting progress of the work and safety of the structures? What are the possibilities of encountering a formation of an entirely different character from anything reported in published survey reports or discovered in the exploratory program?

What are the possibilities of deep decay extending from the surface to tunnel grade, and of large inflows of water? How much timber is likely to be required? When exposed during the excavation of the tunnel, is the rock of any of these formations likely to cause trouble in the form of "popping rock", slaking, disintegration, or oxidation?

Is it likely that great faults or fault zones will be encountered in the tunnel, and, if so, are they likely to give particular trouble either on account of the quality of rock or the quantity of water? Is the rate of progress, due to changes in rock quality, likely to be different in certain sections of the tunnel?

Is there any feature, quality, difficulty, or combination of features that could make the tunnel project impractical; in other words, is the tunnel project feasible from a geological point of view?

Limitations for Suitable Shaft Sites.—The earlier investigational stages were concerned primarily with the location of the tunnel line and the selection of suitable shaft sites. In this connection it was important to recognize the limits of practicability of the information found in published reports, and to distinguish between those areas where the geological conditions could be reasonably and accurately inferred, and those where additional exploratory investigations would have to be made. Of chief concern was the thickness of glacial drift, the depth of buried channels, and their bearing on the location of the tunnel grade. Studies of the topographic map of the area, and more especially of the topographic forms in the field, revealed significant and helpful information which was used to advantage in the studies of tunnel alignment.

The engineering requirements of the project placed certain limits on the location of shafts and tunnel. These may be briefly summarized, as follows:

(1) The easterly terminus of the tunnel (Shaft 1, Fig. 13) was limited to some point adjacent to the Wachusett Reservoir, the westerly terminus (the portal at Shaft 12), to a suitable point along the east side of the proposed Quabbin Reservoir in the vicinity of Quabbin Lake.

(2) An intake shaft (Shaft 8) was to be constructed along the Ware River, near Coldbrook, the location of which was restricted not only by natural and engineering requirements, but also by legal limitations.

(3) The location of a shaft (Shaft 2) was selected near the junction of Trout Brook and the Quinapoxet River, in order that the combined flows of these streams could be diverted westerly for storage in the Quabbin Reservoir at such times as the Wachusett Reservoir was full or was being used for power purposes, if this was found desirable at some future time.

(4) As it was desired to have the easterly section, between Shafts 1 and part way to Shaft 9, completed by the early part of 1931, the tunnel was designed to be excavated from eight shafts, or fifteen headings. Shafts 3 to 7, inclusive, therefore, were located at points where the topography and bed-rock conditions were favorable to their economical construction. The shafts were spaced at as nearly equal intervals as the topography would allow. As finally arranged most of this section was excavated from twelve headings, or six shafts, there being only a small length of tunnel driven from Shafts 1 and 3.

(5) Fewer working headings, and consequently fewer shafts, were necessary for the westerly section as compared with the easterly section, because of the shorter tunnel length, and especially because more time was available for its construction. Because of this fact the westerly tunnel section was laid out to be constructed from only four shafts, or seven headings, Shaft 11-*A* being constructed as a waterway shaft.

PROBLEMS OF THE ROCK FLOOR

The district through which the Quabbin Aqueduct is located is heavily covered with glacial drift and a large part of the bed-rock geology is obscured. Rock outcrops are sufficiently numerous, however, to determine certain major features concerning the underlying geologic structure. Geologic information previously published in Government *Bulletins*[14] was accepted as a working basis. These *Bulletins* proved to be immensely helpful, especially in the beginning of the applied studies; in reality, they became the ready-at-hand guide in the early reconnaissance studies. The scarcity of rock outcrops added to the difficulty of stating the problems in exact and quantitative terms. This feature made it important to re-study the entire related field and re-state the geology in terms suited to the special purposes of the project.

Information gained through geological investigations made it possible to answer, completely, some of the questions previously presented. It was readily determined, for instance, that there were at least seven important geologic units sufficiently uniform within their own boundaries, and yet varying enough, one from another, to be regarded as different geologic formations. It was also possible to determine the formational sequence from one end of the tunnel to the other, and to indicate the approximate distance of the tunnel within each formation. It was impossible, however, to make a definite statement as to the possibility of faulting, as no important faults were discovered in the exploratory work. However, it was necessary to take this feature into account in making an estimate of the amount of ground likely to exhibit unusual behavior, which might require timbering. Because of the crystalline character of the rock as a whole, there was no apparent reason to expect any unusual or rare features that might cause trouble.

It was apparent from the beginning of the project that the tunnel could be constructed successfully along any of the alternative lines as far as the rock formations themselves were concerned. Such structural weaknesses as might be encountered were expected to cross the tunnel line, and these would be encountered no matter what line was followed. The choice of lines was dependent, therefore, not upon the question of rock character, but upon the topography of the rock floor, especially in regard to the depths of buried channels.

SPECIFIC GEOLOGICAL INVESTIGATIONS

Geological investigations included briefly: (*a*) Field work which was required in the construction of an outcrop map; (*b*) delineating areas where the glacial drift was so thick that knowledge concerning the topography and character of the rock floor was uncertain; (*c*) investigation of pre-glacial drainage courses; and (*d*) explorations by borings. Borings to, and into, rock were made in the conventional manner of doing such work. Particular care was taken, however, in being certain of the location of the underlying rock. It was recognized, in this connection, that boulders 5 ft or more in diameter are not unusual occurrences in the glacial drift covering crystalline rock formation. In order to avoid accepting an incorrect impression of the elevation of sound rock, borings were penetrated into ledge for distances varying between 10 and 20 ft. At several of the shaft sites, in addition, the rock formations were penetrated to a depth of 100 ft.

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[14] "Geology of Massachusetts and Rhode Island", by B. K. Emerson, *Bulletin No. 597*, U. S. Geological Survey.

Quinapoxet Valley Investigations.—Great quantities of modified and semi-modified drift deposits occur within the Quinapoxet Valley between Shafts 1 and 2. These deposits form a continuous cover of an uncertain and variable thickness. Such outcrops as were located in the valley indicated that the present stream does not flow in its original channel. Not only was the exact depth of the drift unknown, but it was equally impossible to tell where the deepest channel in the old rock floor should be. A more extensive boring program than usual, therefore, had to be planned to secure the necessary data. The greatest depth of drift penetrated by any of the borings (see Fig. 14) was found to be 114 ft. At points within this valley, there is reason to believe that glacial drift is deposited to depths exceeding 175 ft. Finding the rock floor so low (approximately, Elevation 400) raised serious concern as to the thickness of rock cover over the tunnel as it was then located, and was the chief reason for depressing this section of the tunnel approximately 200 ft. Another indirect reason was that by building the tunnel at this lower elevation, the easterly end could be put under full pressure and could serve as a penstock in the development of power at Shaft 1.

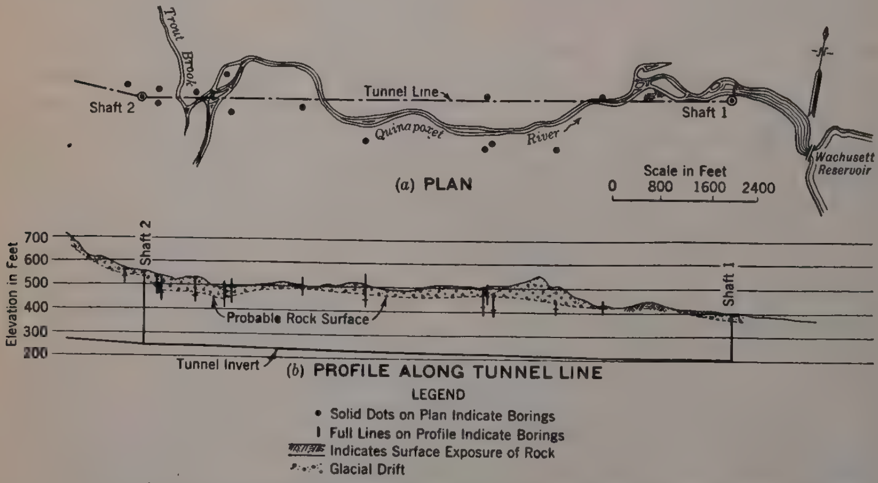


FIG. 14.—LOCATION OF BORINGS AND APPROXIMATE ROCK FLOOR ALONG TUNNEL LINE BETWEEN SHAFTS 1 AND 2, QUINAPOXET VALLEY.

This change of tunnel grade eliminated the question of precise depth of cover and the exact location of the deepest channel, for the greater depth fixed for the tunnel left a large margin of safety. The boring program was thus terminated by a modification of design.

Ware-Prince Valley Investigations.—The abrupt change of the Ware Valley, at Coldbrook, from a north-south to an east-west course, is a striking deviation from the normal trend of valleys throughout the district. One explanation of this behavior is based on the assumption that the Burnshirt-Ware and Prince-Ware River Valleys were originally separate systems. In the early stages of its history, the Burnshirt River flowed by way of the present Burnshirt-Ware Valley as far as Coldbrook, and thence southerly out of the district. The course of the Prince-Ware River at that time was directed the same as at present, but later the Burnshirt-Ware River was diverted into it.

A series of seven borings across the Ware River on the site of the diversion dam at Shaft 8, located the lowest point in the valley floor at about Elevation 607, which is approximately 40 ft lower than the flow line of the present stream. The high elevation of the floor at this point indicates that when the pre-glacial Burnshirt River was diverted, its valley floor was left at an elevation higher than 600 ft above sea level. This conclusion was checked by a boring placed ½ mile southeast of Coldbrook Springs, at a point within the Tertiary Burnshirt Valley, which located rock at an elevation considerably above Elevation 600.

To check the reasoned conclusions concerning the geologic history of these valleys, and also to determine the rock profiles across them, several borings were made near the confluence of the Prince and Ware Rivers. These borings indicated a great thickness of glacial drift over the rock floor. One boring, for example, after penetrating 161 ft of glacial drift showed rock at Elevation 434, which was only a few feet above the projected tunnel grade. That the floor was low at this point had been foreseen, but without further explorations it was not possible to determine whether this condition was confined within the Ware Valley or the Prince Valley, or whether it was common to both. In order to determine the extent of this depression, and to fix the point at which it would be safe to cross the valley along a new tunnel line location, several additional borings were located farther to the east along the tunnel line. These investigations (see profile, Fig. 15) indicated that the rock elevation along the line in the Ware Valley was sufficiently high to assure adequate rock cover over the tunnel, and also that the low floor section is confined to the Prince Valley. It became necessary then to select a crossing under the Prince Valley where a higher rock floor could be found. From the study of the erosional processes, river and glacial, which were responsible for the formation of this valley, it seemed evident that the floor of the valley would be appreciably higher farther north. Consequently, several borings were placed along a new location of the tunnel line and the results confirmed that conclusion. Subsequent investigations indicated a rock valley buried to a maximum depth of approximately 110 ft, with the lowest point of the valley floor at about Elevation 480.

One possible interpretation of the facts disclosed by these investigations, is that the low floor at the confluence of the Prince and Ware Rivers is a result of erosion of the Ware River alone. However, since there is a drop in the rock floor of at least 200 ft in less than 3 miles between Coldbrook and Barre Plains, Mass., it is scarcely likely that the Ware River alone could have been responsible for such an abnormal gradient. Further explorations within the Ware Valley between Barre Plains and Coldbrook, showed rock to be considerably higher than would have been expected had the valley been eroded solely by the river. The most reasonable interpretation seems to be that local over-deepening by glacial scour is responsible for the low rock floor in the vicinity of Barre Plains.

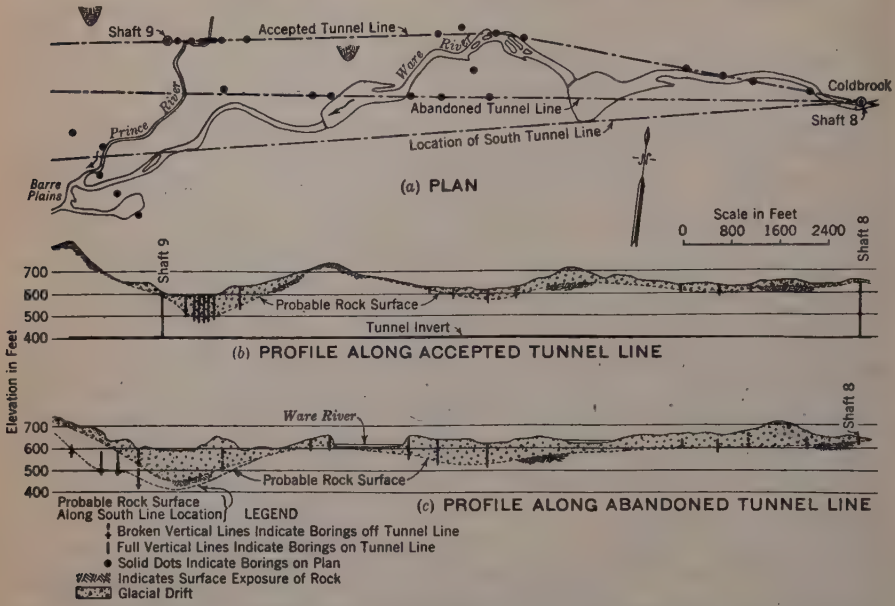

Shaft 9
Accepted Tunnel Line
Ware River
Prince River
Abandoned Tunnel Line
Location of South Tunnel Line
Coldbrook
Shaft 8
Barre Plains
(a) PLAN
N
Scale in Feet
0 800 1600 2400
Shaft 9
Shaft 8
700
600
500
400
Probable Rock Surface
Tunnel Invert
(b) PROFILE ALONG ACCEPTED TUNNEL LINE
Elevation in Feet
Ware River
Shaft 8
700
600
500
400
Probable Rock Surface
Probable Rock Surface Along South Line Location
(c) PROFILE ALONG ABANDONED TUNNEL LINE
LEGEND
Broken Vertical Lines Indicate Borings off Tunnel Line
Full Vertical Lines Indicate Borings on Tunnel Line
Solid Dots Indicate Borings on Plan
Indicates Surface Exposure of Rock
Glacial Drift

FIG. 15.—RELATION OF ALTERNATIVE TUNNEL LOCATIONS, TOGETHER WITH DISTRIBUTION OF EXPLORATORY BORINGS AND PROFILES, WARE-PRINCE VALLEYS.

Muddy Brook Valley Investigations.—Muddy Brook Valley is a fine example of a large valley now occupied by a stream apparently much too small to have accomplished such an extensive erosional effect. For more than $1\frac{1}{2}$ miles, the broad floor of this valley is covered with low sand and gravel ridges of glacial origin. Most of the drift is assorted and could be accumulated to almost any depth. There is nothing whatever to be seen in the valley bottom, from surface inspection, that would indicate either the form of floor profile or the quality of rock beneath it.

Preliminary investigation showed an irregular distribution of glacial drift, and suggested the conclusion that Muddy Brook Valley was the result of preglacial stream erosion, but that considerable deepening and widening may have been accomplished by glacial abrasion. How much of an effect these processes had on the final configuration of the rock floor it was impossible to foretell.

The first boring explorations in the valley were made along a trial location designated as the south line. A rock profile developed from several borings placed across a part of the valley near this line revealed a deeply buried valley. The lowest point of the floor along this line was determined at approximately 45 ft above the tunnel roof. Because of the relatively thin rock cover through here, attention was next focused upon a more northerly location of the tunnel, where conditions were believed to be slightly better. However, the results showed a continuous low floor, and also that the present divide in this area, between the Ware and Swift River water-sheds, is formed entirely of glacial drift deposited to a maximum depth of at least 140 ft. As a result of these studies, which indicated a rock cover over the tunnel roof of about 55 ft, the north line was accepted as the tunnel location.

Upon reviewing the entire Muddy Brook problem at a later date, it was felt that there were still several features of the problem which had not been as fully determined as was desirable. Borings on the north line had been located by lining up the lowest known part of the valley floor as developed on the south trial line with the probable water gaps to be observed in the topography toward the northwest. This method, however, left an unexplored portion along the line approximately 1 mile in length. This was much too long a stretch to be taken wholly for granted.

For the particular work of re-exploration, it was considered practicable to develop a rock profile with the aid of the electrical resistivity method of geophysical prospecting. Estimates of the thickness of cover or depth to bedrock were based on electrical measurements. In order to be assured of the approximate reliability of the results, similar electrical determinations were made at two points where the thickness of cover was already known from borings. In both cases the concordance of results was close.

A rock profile was thus developed across the valley floor, which considerably extended the reliable information. In Fig. 16, the profile is drawn along the north line (the final location) and defines the approximate rock floor as indicated by borings and geo-physical explorations. The group of borings farther south were located on a preliminary trial line which was abandoned in favor of a location farther north. Throughout the re-explored section in question, Fig. 16(*b*), the mantle of glacial drift was shown to be rather thick, but the elevation of the rock floor was considerably higher than in the adjoining section on the west. As a result of these investigations, successful excavation of the tunnel through the rock floor of this valley was made more certain.

Portal Site Investigations.—The first steps in the sub-surface investigations of the west portion of the Quabbin Aqueduct were made in Muddy Brook Valley, along a line referred to as the "south line." In the beginning, location studies of the west terminus of the tunnel were confined to the selection of a suitable portal site on the East Branch of the Swift River, somewhere south of Quabbin Lake. The portal of the south tunnel line was located about $\frac{1}{4}$ mile south of the lake at a natural site. It was noted, however, that the terrain along the east side of the lake would allow for considerable saving, in the length of tunnel—about $\frac{1}{3}$ mile—for a portal located farther north. Preliminary studies narrowed the list of possible sites to two,

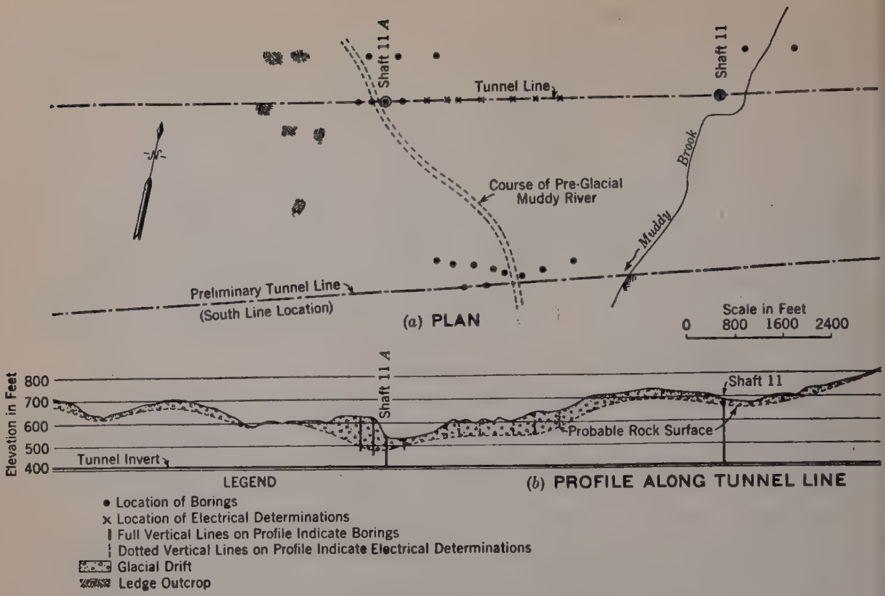


Fig. 16.—Borings in Muddy Brook Valley.

one at the extreme north end of Quabbin Lake, and the other midway along the east section. After careful investigations by borings, the most northerly site was finally chosen as the portal. This choice made it possible to take advantage of the shortest possible tunnel location with its consequent saving in construction cost.

Tunnel Design and Location Adopted

As finally located, the Quabbin Aqueduct is oriented in a general east-west direction and extends from the northwest side of Wachusset Reservoir to Quabbin Lake (see Fig 13). Excavation was carried through rock for its entire length of 24.6 miles. The quantity of rock cover above the tunnel, as nearly as could be determined by borings, varies between 55 and 800 ft. In the easterly 3½ miles, the tunnel level or invert lies between Elevations 200 and 385. For the remaining length it continues in a slightly inclined grade to the portal, where the invert is at Elevation 414. All elevations given herein and on the diagrams refer to the Boston City Base datum plane.

The completed tunnel will serve as an aqueduct which, because of the surface topography of the area and its design, will fulfill several functions: (*a*) It will furnish a means of bringing water from the Quabbin Reservoir which will have a maximum flow-line at Elevation 530; (*b*) at Shaft 8, at Elevation 656, it will permit diversion by gravity of the Ware River into the Quabbin Reservoir, or the Wachusett Reservoir; and (*c*) at Shaft 2, at Elevation 540, it will permit diversion of a considerable part of the Quinapoxet River flows into the Quabbin Reservoir if, and as, this is desired. The tunnel flow controlled by gates at the east, or the Wachusett Reservoir, end will normally enter the Quabbin Reservoir through Shaft 11-*A*, located about 2½ miles east of Shaft 12. When the flow is reversed, water from the Quabbin Reservoir will enter the tunnel from Shaft 12 and flow eastward into the Wachusett Reservoir.

The Quabbin Aqueduct is not a pressure tunnel as this term is commonly understood in the eastern part of the United States. For about 500 ft at the easterly end, and for about 200 ft at the westerly end, the tunnel is circular in section, having an inside diameter of 12 ft 9 in. In these two stretches the concrete lining was grouted solidly to the rock, under high pressure. Between these circular ends, which behave as plugs in preventing leakage from or into the tunnel, the tunnel section is of the horseshoe type, 12 ft 9 in. high by 11 ft wide. The concrete lining is only of sufficient thickness to insure stability in the various types of rock in which the tunnel is built. Weep-holes were constructed in the bottom, arch, and elsewhere, as frequently as it was deemed necessary. These weep-holes prevent excessive hydrostatic pressure upon the masonry lining during operation as a result of rapid changes in the hydraulic gradient. Pressure from the tunnel outward is carried through the weep-holes into the surrounding rock, and water pressure from the rock inward is relieved by flow through the weep-holes as the hydraulic gradient drops.

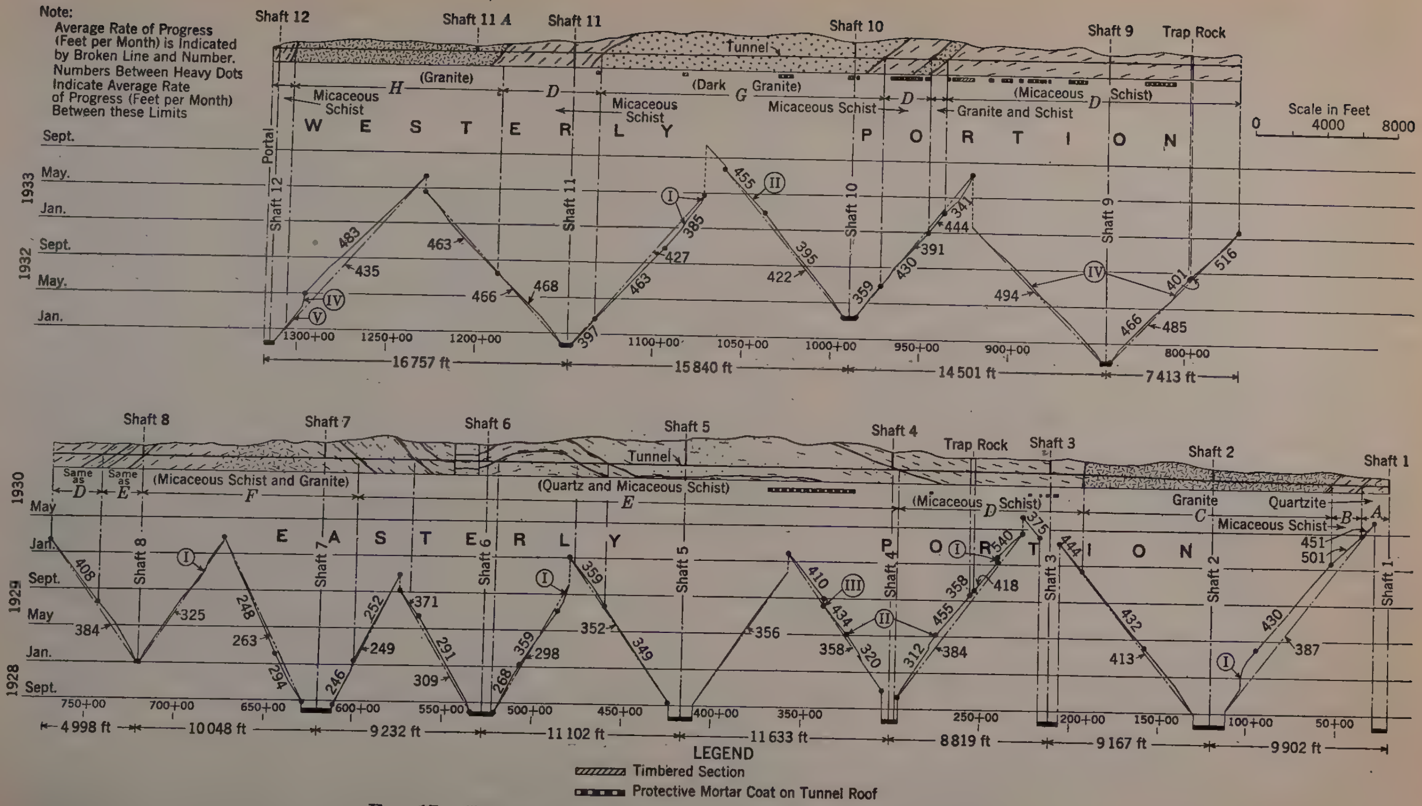
The tunnel portion from Shaft 8 to the Wachusett Reservoir, about 14 miles in length, has been operated under extreme conditions at intervals since 1931 without any signs of structural or other defects.

To determine the payment for rock excavation in the tunnel, certain reference lines were provided and designated as the *A* and *B* lines. The *A*-line was defined as that within which no excavated material of any kind was permitted to remain. In the normal tunnel cross-section, this line was placed at a minimum distance of 3 in. outside the neat line of the concrete side-walls, and of 8 in. above the neat line of the arch. The *B*-line was arbitrarily placed 12 in. outside the *A*-line in the side-walls and arch, and 2 in. below the *A*-line in the invert. Slight modifications of these distances were made where permanent timbering was required. All payment for excavation in the untimbered portions was for the volume bounded by the *B*-line, or normally about 6.8 cu yd per lin ft.

All shafts, except Shafts 1, 8, and 12, had an inside diameter of 14 ft. The three exceptions were larger than this and were designed to serve not only for construction purposes, but also as waterway shafts. Shaft 1 was excavated as one shaft, but completed as two units, one unit being used as a water outlet, the other as a pump-well for dewatering the depressed tunnel section between Shafts 1 and 3.

TUNNEL EXCAVATION

Rock Formations Encountered.—From an engineering viewpoint, the rocks of the tunnel area can be divided into two major classes, schist and granite. Both these varieties are highly crystalline, the granite being of an igneous origin, and the schist being originally of sedimentary origin. Excavations



Note:
Average Rate of Progress (Feet per Month) is Indicated by Broken Line and Number. Numbers Between Heavy Dots Indicate Average Rate of Progress (Feet per Month) Between these Limits

LEGEND
▨ Timbered Section
▬ Protective Mortar Coat on Tunnel Roof

FIG. 17.—PROGRESS OF TUNNEL EXCAVATION AND GEOLOGICAL SECTION.

were made through two belts of an average variety of granite (see Fig. 17) which occur in the easterly and westerly portions of the tunnel. The rock in these sections is not all intrusive granite in the true sense of the term, as incorporated masses of a schistose rock also occur. Extensive portions of one of these formations, in addition, are composed of a different and harder variety of igneous rock known as diorite. A third granitic formation occurs throughout the middle portion of the westerly tunnel section which is darker, more massive, and slightly harder than the other two average varieties.

The granite formations occupy about one-third the bulk of rock excavated from the tunnels. The remaining two-thirds is composed of laminated beds of schist, which originally constituted a thick series of sedimentary rocks, composed of sandstone and shale. These schistose rocks have been greatly modified by igneous intrusions and widespread impregnations of granitic constituents. Although the schists have been divided into five distinct formations, they are essentially of two types. One of these is a generally soft micaceous schist, and the other a harder flaggy quartz schist. Diabase or trap-rock dikes cut the micaceous schist formations at two widely separated points, but the distance tunneled through them was comparatively short.

The rocks excavated in the tunnel are common types, and occur over large areas not only in New England, but elsewhere. Their counterparts may be traced throughout a large part of the Piedmont belt extending from New York State to Alabama. Similar types constitute entire formations in other sections of the United States where sedimentary sandstone and shale adjacent to large intrusive igneous bodies have been modified by the injection of igneous matter.

Scaling Rock.—In general, the quality of the rocks was found to be highly suitable for tunneling operations. A few cases of unusual rock behavior deserve comment, however. Excessive scaling in the arch, throughout much of the West Heading of Shaft 4 created at first a serious problem from the standpoint of safety and maintenance of excavation progress. The cause for this behavior was due to what is generally known as "air slacking." This condition developed with time, for when the difficulty first attracted serious attention the easterly part of this section had been exposed to the air for about three months.

Several factors of a geologic nature contributed to the final result. The quartz schist rock throughout this heading is finely laminated because of the occurrence of mica streaks, and is arranged in thin beds having a slight easterly dip. It is broken into large and small angular blocks and slabs by joints, which are covered with a thin coating of carbonate minerals, chiefly calcite, associated with iron sulfide minerals. Exposure to air for a period, oxidized the sulfide and softened the carbonate minerals. Due to this behavior, the bond between separate blocks was loosened which resulted in large slabs dropping from the roof section. This condition was combatted successfully, without timbering, by the application of two, and sometimes three, coats of Portland cement mortar in sufficient thickness to seal the joints and other cracks from contact with air.

The granite, throughout a short stretch in the easterly tunnel portion, also scaled, but this behavior resulted from a geologic condition which differed from that causing "air slacking." The grain of the granite and the orientation of incorporated masses of schist indicate a strong geologic structure, the rock lying in a nearly vertical position. The entire formation was under strain which, upon being released by excavation of the tunnel, developed new fractures causing slabs to fall from the arch. This condition commonly known as "popping rock" required constant attention and the frequent scaling of rock in the arch by the contractor. Covering the arch section with mortar would not have improved the situation in this case, and the condition was not sufficiently serious to require timbering.

Timbering.—A question of practical importance, always raised on a project of this kind, concerns the amount of timbering likely to be required, and which particular sections of the tunnel are most likely to require such construction.

No definite answer could be given in advance to this question, but it was thought that the rock would require timbering throughout possibly a maximum of 10% of the tunnel. In the estimates it was assumed that about 5% would require some form of timber support. The actual figures, including those for correcting the condition of "air slacking" in the arch, are as follows:

	Feet		Percentage
Permanent timber support.........	1 971	....	1.5
Temporary timber support.........	320	....	0.3
Protective mortar coat............	12 050	....	9.3

Many intrusions of granite in the micaceous schist formations established conditions that were followed by alteration and gradual decay of the adjacent rock. Partings along bedding lines served as watercourses, so that the entire formation was affected, some sections more so than others. This condition caused the rock to scale and crumble, which made it necessary to construct timbering throughout the stretches where the formations were softest. The protective coat was sufficient to stabilize the rock in other sections where alteration and consequent decay were not so prominent.

Excavation was carried through several zones of crushing and close jointing without any abnormal rock behavior. At some points, however, overbreakage on the sides and in the arch resulted from such conditions. Timbering was constructed in some of these sections, especially where the removal of loose blocks did not appear to remedy the unstable situation. In a short stretch, near Shaft 12, the rock had a tendency to swell and move into the excavated space. This behavior was due to the occurrence of large quantities of clay derived by alteration of an intrusive igneous body containing much feldspar.

PROGRESS OF EXCAVATION IN RELATION TO GEOLOGIC CONDITIONS

Schedule of Required Progress.—The first contracts, awarded in 1927, were those for the construction of Shafts 2 to 7 in the easterly tunnel portion. The award of two major contracts during the latter part of 1927, for constructing the easterly tunnel section from and including Shaft 1, to 4 000 ft beyond and including Shaft 8, was delayed about five months by unforeseen legal difficulties. During this time the construction of six shafts was completed, and a considerable length of tunnel, in all 8 238 ft, was excavated with equipment used in sinking the shafts. The schedule of minimum progress in the first major contracts, required that excavation of 65 664 ft of tunnel should be completed within 26 months. The third major contract, for constructing the westerly portion of the tunnel, including four shafts and appurtenant structures, was awarded during the early part of 1931. The schedule of progress on this contract required all excavation in shafts and 54 845 ft of tunnel to be completed in 34 months. The contract requirements as to the minimum performance in excavating these tunnels may be summarized as shown in Table 6. As actually constructed, all but 836 ft of the tunnel between Shafts 1 and 2 was excavated from Shaft 2, and the actual length of tunnel driven west of Shaft 8 was 5 000 ft. Furthermore, after being excavated, Shaft 3 was not used extensively as a construction shaft, and practically all the heading between Shafts 3 and 4 was excavated from Shaft 4.

TABLE 6.—CONTRACT REQUIREMENT FOR MINIMUM PERFORMANCE

Shaft (see Fig. 13) (1)	Length of tunnel to be excavated, in feet (2)	Minimum requirement, in feet per month* (3)
(*a*) EASTERN PORTION		
1†	4 000	275
2‡	5 034	220
2 to 3	7 634	330
3 to 4	7 811	340
4 to 5	10 561	460
(*a*) EASTERN PORTION (*Con.*)		
5 to 6	9 734	425
6 to 7	7 702	335
7 to 8	9 188	440
8†	4 000	275
......	....	....
(*b*) WESTERN PORTION		
9‡	7 414	260
9 to 10	14 501	550
10 to 11	15 840	620
11 to 12	16 757	580
12§	319	....

* Minimum progress required in one or more headings to complete tunnel excavation in the specified time. † West Shaft. ‡ East Shaft. § Shaft 12 to West Portal.

The rates indicated in Table 6, especially those in Table 6(*a*), were estimated from the average performance attained on other tunnel projects throughout the eastern part of the United States. Those in Table 6(*b*) were based upon the average performance attained in excavating the eastern part of the Quabbin Aqueduct with liberal allowance for unforeseen construction difficulties. A few of these expected difficulties, which are essentially of a geologic nature, and of which complete information could not be obtained in advance may be thus summarized:

1.—As the tunnel crosses the general trend of several geologic formations, over a belt 25 miles wide, it was expected that some of the headings would pass through several faults and crush zones requiring timber support and in other ways creating serious difficulties. The results of studies of surface outcrops and core borings, made at critical and intermediate points, did not reveal the occurrence of such zones of weakness. It was realized, however, that this was not unusual because of the extent to which glacial drift deposits had buried the underlying rock formations. It was believed expedient, therefore, to allow for the possibility that this condition might occur at least as frequently as once in every mile or so.

2.—An allowance was also made for possible occurrence of deep decay extending from the surface to the tunnel grade, or disintegration or slacking of the rock when exposed to air. The least troublesome of these conditions would require only precautionary measures during excavation, and if they were sufficiently extensive, timbering, cement mortar lining, or some other form of construction.

3.—The occurrence of rocks of a crystalline character was recognized, and also that they would not contain water, except within the space between joints, and other fractures and fissures. The tunnel passes under several river valleys and areas containing porous glacial deposits, and considerable leakage into the tunnel from these was anticipated, together with the attendant hazards of inflows temporarily beyond pumping facilities.

4.—Changes were made in the original location of the aqueduct in order to cross under two deeply buried valleys. The possibility of encountering serious difficulties in these stretches, because of the presence of faults or crush zones, deep decay, large inflows of water, or because other combinations of adverse considerations made it expedient to allow for considerable retardation of progress.

Methods of Excavation.—A description of the methods used in tunnel excavation is included in order to supplement the information presented in Fig. 17 and Table 7. The data concern only the performance attained after permanent tunnel equipment had been installed. The shorter tunnel stretches excavated with the equipment used in shaft sinking are not considered. Shaft 3 was eliminated as a construction shaft soon after construction began under the first major tunnel contracts, but access to it was maintained as insurance against difficulties encountered in excavating from Shaft 2. Only a short stretch of tunnel was excavated from Shaft 1.

The numerals within circles on the progress chart in Fig. 17 refer to geological or other conditions which appreciably affected the performance of the work, as follows: (I) Progress retarded by water; (II) increase in progress due to changes in organization; (III) progress retarded by "air-slacking" rock; (IV) progress retarded by unstable rock, requiring timbering; and (V) slow progress due to experiments on the dragline mucking machine.

The full-face method of excavation was used at Shafts 2, 4, and 9 to 12, inclusive. The headings at the first two shafts were advanced continuously by drilling and mucking crews, which worked on a schedule of three 8-hr shifts. By this plan the contractor hoped to obtain four advances in one day and five the next. This objective was realized at times, but not to the extent anticipated. Two 10-hr shifts were employed at Shafts 9 to 12, inclusive. The number of advances in one day was seldom less than four, and five advances per day were attained at times, except at Shaft 12, where there was one heading. Three advances were often made at Shaft 12, but in order to do this, the crews were required to work overtime.

TABLE 7.—DATA RELATING TO EXCAVATION AND CHARACTER OF ROCK

Description	Quartzite	Micaceous schist	Granite	MICACEOUS SCHIST			QUARTZ SCHIST		Micaceous schist and granite	DARK GRANITE		GRANITE	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(a) PROGRESS DATA													
Location in Geologic Section (see Fig. 17)	*A*	*B*	*C*	*D*	*D*	*D*	*E*	*E*	*F*	*G*	*G*	*H*	*H*
Method of excavation	*	*	*	*	*	*	*	†	†	*	*	*	*
Heading in Which Formation Occurred; Shaft	2‡	2‡	2§	4‡	9§	11§	4§	5‡	8‡	10§	11‡	11§	12‡
Average Advance per Shot, in Feet: Monthly	7.3	7.7	7.0	6.5 and 7.3	8.8	8.3	6.8 and 7.3	6.2	6.5	7.3 and 7.8	7.8	8.0	7.7
Range: From	7.1	7.4	6.1	6.1‖	7.2	6.6	6.3¶	5.8	5.8	6.8**	6.0	7.6	7.0
To	7.4	7.9	7.4	6.9	9.5	9.4	7.8	6.7	7.5	8.2	8.5	8.8	8.9
Number of Days in Which There Was no Advance, for the Following Reasons: Timbering	0	0	0	5	7	0	0	0	0	0	8	0	0
Poor roof	0	0	0	2	0	0	25	1	0	2	0	0	0
Water	0	0	0	18	0	0	0	0	32	0	4	0	0
Holidays	0	0	2	4	5	5	4	2	2	5	4	0	4
Repairs to plant, etc.	0	0	3	7	7	4	11	2	2	4	3	0	6
Number of Shots Daily: One-round shots	4	2	51	75	35	24	102	36	62	61	38	17	23
Two-round shots	37	76	362	350	392	216	285	393	306	497	372	252	300
Three-round shots	7	21	37	54	4	0	30	0	0	0	0	0	78
(b) DRILLING AND BLASTING DATA													
Range in Number of Holes in Heading: From	30	31	30	30	38	29	32	24††	26††	36	31	29	30
To	32	35	40	40	40	36	40	32††	34††	44	36	36	42
Typical heading: Number of holes	31	31	37	37	38	32	37	28†† 6‡‡	29†† 7‡‡	40	32	36	36
Average total drilling, in feet	287	287	345	345	386	346	345	224†† 48‡‡	244†† 56‡‡	440	288	396	396
Average Depth of Individual Holes, in Feet: Cut	9	9	9	9	9 and 11	10+	9	8	10	9 and 11	9+	9+	9 and 11+
Relief	8 and 9	8 and 9	8 and 9	8 and 9	9 and 11	10	8 and 9	8	8	9 and 11	9	9	9 and 11
Trim	8 and 9	8 and 9	8 and 9	8 and 9	9 and 11	10	8 and 9	8	8	9 and 11	9	9	9 and 11
Speed of Drilling, in Inches per Minute per Drill: Typical	7	10	7	12	16	13	9	9	11	6	6	7	7
Ranging: From	5	5	5	7	10	7	7	7	7	5	5	5	5
To	9	12	9	15	20	20	11	11	14	12	12	9	9
Number of drills at heading	5	5	5	5	4	4	5	2	2	4	4	4	4
Average Time Required to Drill a Heading, in Hours: From	2	1.5	2	1.5	1.5	2	2	5††	5††	3	3	2.5	2.5
To	3.5	3.5	3.5	2.5	2.5	3	3	7††	7††	4	4	3.5	3.5
Average Time Required for Drilling - Blasting Cycle, in Hours: From	4	3	4	3	2.5	3	3.5	9	9	4	4	3.5	3.5
To	5	4	5	4	3.5	4	4.5	10	10	5	5	4.5	4.5
Average Unit per Cubic Yard of Excavation: Linear feet of holes	6.0	5.7	7.0	7.0	6.5	6.5	7.2	6.5	6.5	7.0	6.7	6.5	7.3
Pounds of explosive	5.0	4.5	6.5	6.0	3.8	4.9	6.2	6 0	6.0	5.7	6.0	5.3	5.1

* Full face. † Top heading and bench. ‡ East of shaft. § West of shaft. ‖ Also from 6.7 to 7.8. ¶ Also from 6.7 to 7.9. ** Also from 7.5 to 8.2. †† Top heading. ‡‡ Bench.

The heading and bench method was used in the headings of Shafts 5 to 8, inclusive. At these points the work was organized on a basis of two 10-hr shifts in such a manner as to produce two rounds of excavation per day in each heading. This arrangement required drilling and mucking to be done together.

At Shaft 7 a different plan was used in which drilling and blasting of each heading and bench was completed during one shift and mucking during the other. The depth of holes was increased to 10 ft. The heading was blasted in two operations, the cut and bench holes being loaded and shot first.

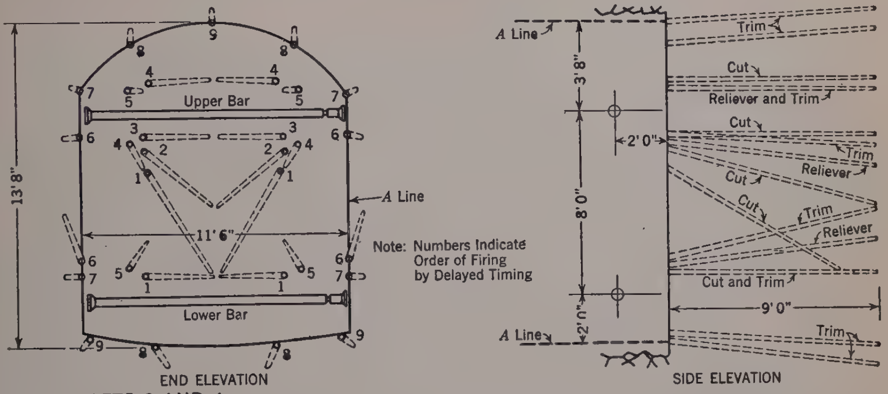
Drilling and Blasting.—Holes were drilled in the full headings in much the same manner as on similar projects elsewhere. Two types of drill carriages were used. The drilling bars on one of these carriages were integral parts of the carriage, whereas those on the other were separate from the carriage and had to be blocked and wedged to the side-walls. Four drills, one on each bar, were mounted on the former carriages whereas five drills—three mounted on the upper and two on the lower bars—were used on the latter. Three drilling and three mucking crews, each working in an 8-hr shift, operated between the headings at Shafts 2 and 4. In the westerly tunnel section, a similar drilling and mucking organization was used, except that the crews worked on only a 10-hr per shift schedule.

As good, and in several cases even better, progress was attained in excavating the easterly section, under similar rock conditions. The integral type of drill carriage, compared to the other type, proved to be a time-saver and in this way contributed toward a better performance record. Holes in the top headings were drilled by two machines mounted on horizontal arms clamped to two vertical columns, which were blocked and wedged to the tunnel roof and bench. Bench holes were drilled by the same machines, mounted on one horizontal bar located in front of the bench, and wedged to the tunnel side-walls. Drilling and mucking were simultaneous operations in these headings.

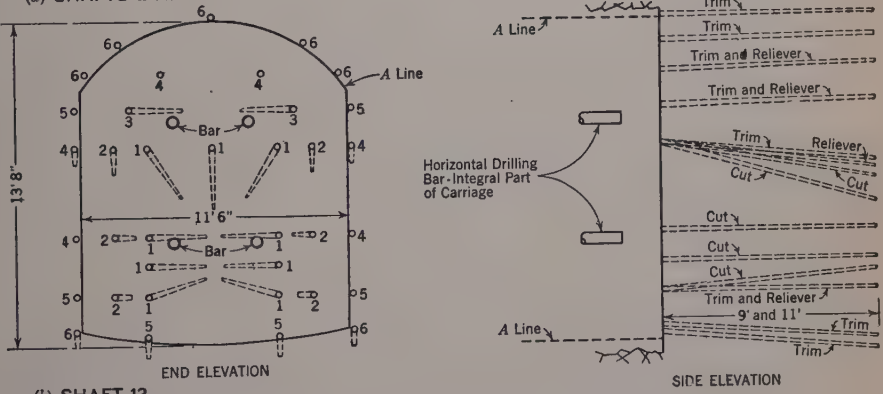
The location and pointing of drill holes is illustrated in Fig. 18. A typical 31-hole drilling round, used in the granite and schist formations of Shafts 2 and 4, is shown in Fig. 18(*a*). In the harder varieties of granite, the number of holes was increased generally to 37, and in the softer varieties and throughout long stretches of micaceous schist, the number was reduced to 31 holes, or less. A similar diagram showing 36 drill holes (the number generally used in the granite formation of Shaft 12) is shown in Fig. 18(*b*). The variation in the distribution of holes, is due to several reasons, chief of which is the difference in locations of the drilling bars on the two drill carriages.

In blasting, the center or cut holes in both the full-face and top headings were generally loaded with 60% gelatine dynamite, and the remaining holes with dynamite of 40% strength. At times, dynamite of 40% strength was used in all holes, especially in stretches of soft rock. Throughout the westerly tunnel portion the contractor used a new explosive product, in strengths of 40, 47, and 60 per cent. The quantity of explosive used per cubic yard of excavation in the different kinds of rock is indicated in Table 7.

Mucking.—In all the full headings, except the single one at Shaft 12, blasted rock was loaded by means of electric power shovels. At Shaft 12, the contractor installed a dragline excavator of his own construction. This machine was equipped with a conveyor belt of sufficient length to allow a full train of five cars to be loaded without the necessity of switching and by-passing cars. With the power shovels, empty cars were moved from the front to the rear end of the train, nearest the loader, by means of an air hoist or car transfer. Hand-mucking was used for a time in the headings where the top-heading and bench method of excavation was followed, but this was abandoned later in favor of mechanical mucking by air shovels.



(*a*) SHAFTS 2 AND 4



(*b*) SHAFT 12

FIG. 18.—LOCATION OF DRILL HOLES IN FULL HEADINGS.

The time required to remove rock from the headings varied. In the full headings, except those at Shafts 11 and 12, the average time was between 2.5 and 3.5 hr. At Shafts 11 and 12 it was between 3 and 5 hr. Where the top-heading and bench method was used the average time was between 5.5 and 6.5 hr.

Throughout the easterly portion, loaded muck cars were hoisted from the tunnel by means of cages, whereas, in the westerly portion, skips were used. Provision was made at Shafts 9 to 12, inclusive, for stocking rock of suitable quality and size for use as concrete aggregate. This was accomplished by wasting rock in those headings where the formations were soft, and by dumping rock from hard formations over a coarse screen or "grizzly." The portion passing through this separator was then crushed and conveyed by belt to the stock-pile. From experience on the Quabbin project, the method of stocking suitable rock material and wasting that of inferior quality is believed to be superior to that of dumping excavated rock on a spoil bank regardless of quality. This is especially true where tunnels are excavated through different geological formations, some of which may be soft or in other respects may have objectionable characteristics.

Hardness of Formations.—The relative hardness of the rocks is shown in Table 7(*b*) under the heading, "Speed of Drilling." A wide range of drilling behavior is indicated. The micaceous schist formations were the softest, the high mica content and soft condition, due chiefly to alteration of minerals, being conducive to fast drilling. Slower drilling in these formations was due to encountering quartzite beds and granite bodies.

The hardest rocks to drill, except trap-rock, were those contained in the granite formations. In general, the granitic rocks were about three times as hard to drill as the micaceous schists. Between the extremes represented by these formations there was a rather uniform hardness represented in the flaggy quartz schist types and the micaceous schists containing much granitic material. The widespread intrusions of granite in portions of the micaceous formations rendered the rock harder and more stable than would otherwise have been the case.

EXCAVATION COSTS

Information pertaining to the cost of excavation in the several rock formations is shown in Table 8. Partial information relating to this particular item has been published[15] previously, although not in the form presented herein. The values given are the costs for excavating to approximate line and grade only. Before lining, the contractors were required to remove rock which projected beyond the arbitrary limits on the side walls and the arch, and also to shape the tunnel invert to grade. The cost of this operation is not included in Table 8.

The much lower cost indicated in Column (10), Table 8(*b*), is due mostly to the use of second-hand equipment, purchased from the contractors who constructed the easterly portion. This equipment, which had been maintained in excellent condition, was obtained at greatly reduced prices, compared to the original costs. Comparison of costs in the easterly tunnel portion (Table 8(*a*)) with those in the westerly portion (Table 8(*b*)) indicate striking differences

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[15] "Construction of Wachusett-Coldbrook Tunnels", by Douglas C. Corner, U. S. Bureau of Mines; "Boston Metropolitan Water Supply Extension", by Karl R. Kennison. M. Am. Soc. C. E., *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2.

TABLE 8.—Approximate Operating Cost of Excavation, Quabbin Aqueduct, After the Installation of the Permanent Tunnel Equipment

(Values in Parentheses Are Dollars per Cubic Yard; the Remaining Values Are Dollars per Linear Foot)

Item No. (1)	Shaft (see Fig. 13) (2)	Rock formation (see Fig. 17) (3)	Location (see Fig. 17) (4)	Labor cost for all conditions, geologic or otherwise (5)	Labor cost for average conditions in good rock (6)	Labor cost for both headings for all conditions, geologic or otherwise (7)	Power (8)	Explosives and detonators (9)	Material and supplies (plant, etc.) (10)	Cost of overhead (11)	Total operation cost (12)
(a) Eastern Portion											
1	2	Quartzite........	*A*	$23.80 (3.50)	$23.80 (3.50)	$27.50 (4.05)	$5.10 (0.75)	$6.20 (0.90)	$11.30 (1.65)	$4.10* (0.60)	$54.20 (7.95)
2	2	Micaceous schist	*B*	21.70 (3.20)	21.70 (3.20)						
3	2	Granite........	*C*	28.20 (4.15)	24.30 (3.60)						
4	2	Micaceous schist	*D*	24.30 (3.60)	23.60 (3.45)						
5	4	Micaceous schist	*D*	29.10 (4.28)	23.70 (3.50)	$32.50 (4.80)	$6.00 (0.90)	$5.60 (0.80)	$14.80 (2.15)	$3.80* (0.55)	$62.70 (9.20)
6	4	Trap-rock......	......	32.80 (4.85)	32.80 (4.85)						
7	4	Quartz schist....	*E*	33.20 (4.90)	27.80 (4.10)						
8	5	Quartz schist....	*E*	28.20 (4.15)	26.10 (3.85)	$28.20 (4.15)	$4.40 (0.65)	$5.80 (0.85)	$8.90 (1.30)	$2.10* (0.30)	$49.40 (7.25)
(b) Western Portion											
9	9	Trap-rock......	......	$20.50 (3.00)	$20.50 (3.00)	$16.00 (2.35)	$2.80 (0.40)	$3.70 (0.55)	$5.50§ (0.80)	$4.40§ (0.65)	$32.40 (4.75)
10	9	Micaceous schist	*D*	15.60 (2.30)	14.10 (2.10)						
11	10	Micaceous schist	*D*	17.90 (2.60)	14.50 (2.15)	$18.10 (2.65)	$5.00 (0.75)	$4.80 (0.70)	$6.10§ (0.90)	$4.60§ (0.65)	$38.60 (5.65)
12	10	Dark granite....	*G*	18.20 (2.70)	15.40 (2.25)						
13	11	Dark granite....	*G*	16.90 (2.50)	14.70 (2.15)	$15.90 (2.35)	$4.10 (0.60)	$4.50 (0.65)	$5.50§ (0.80)	$4.40§ (0.65)	$34.40 (5.05)
14	11	Micaceous schist	*D*	13.90 (2.05)	13.90 (2.05)						
15	11	Granite........	*H*	16.10 (2.35)	16.10 (2.35)						
16	12†	Granite........	*H*	21.30‡ (3.15)	17.50‡ (2.60)	$21.30‡ (3.15)	$4.90 (0.70)	$4.90 (0.70)	$9.10§ (1.35)	$9.00§ (1.35)	$49.20 (7.25)

* Taken from *Information Circular*, entitled "Construction of the Wachusetts-Coldbrook Tunnel", U. S. Bureau of Mines. † Cost data for one shaft only. ‡ Labor costs of excavation after development of dragline excavator and after passing through a section that required timbering. § More approximate than other cost data.

of excavation costs in formations of similar rock character and behavior. This may be partly accounted for, as follows:

(1) The easterly tunnel portion was excavated during 1928–30, and the westerly portion during the period, 1931–33. During the latter period, the wage scale of the several classes of labor was from 5% to 25% lower than during the former years.

(2) Better progress as a whole was made in the westerly portion, where the full-face method of excavation was used entirely, than in the easterly portion, where only one-third of the headings were excavated by the full-face method and two-thirds by the slower top-heading and bench method.

(3) The number of man-hours per day, in the easterly portion, was generally from 10% to 20% greater than those in the westerly portion.

(4) Micaceous schist formations extended through longer stretches of the westerly tunnel portion than of the easterly portion. In addition, these schists were considerably softer than their counterparts farther east. These two conditions are partly responsible for the better progress throughout the westerly portion.

(5) Disregarding the single heading at Shaft 12, better progress was made in the granite formations of the westerly portion as compared to that made in granite of similar quality in the easterly portion. This advantage was largely the result of longer advances per shot, which, in turn, were due to an improved type of explosive, and deeper, and possibly better directed, drill holes.

Rock Breakage.—High and wide over-breakage at several places was caused by closely spaced and intersecting joints. This condition had some effect upon the actual volume of rock excavated as compared to the volume within the theoretical payment lines. The payment line for excavation was fixed arbitrarily 12 in. outside the *A*-line, the mimimum required line of excavation in the side-walls and arch. As excavated, the average rock breakage beyond the *A*-line, considering the total length of tunnel, was about 10½ in.

CONCLUSIONS

An attempt has been made in this paper, to correlate data of a geologic nature with that related to engineering and construction problems. Construction of the Quabbin Aqueduct, without encountering serious difficulties, may be attributed partly to the sound character of the rock formations through which the tunnel extends. It can be emphasized, however, that many of the difficulties that might have been encountered were eliminated by careful geologic investigations, which reduced construction uncertainties to a minimum, and conversely determined the favorable geologic factors.

Several items of interest may be emphasized in conclusion, as follows:

1.—The kind of geological investigations described herein are also applicable to the study of similar problems in tunnel location in other sections of the United States. Valleys which have been deeply buried by glacial drift deposits are not new topographic features to the engineer. Numerous accounts of the crossing of such valleys by tunnels may be found in engineering literature. In presenting details concerning the surface geologic features of the

Quabbin Aqueduct the writer wishes to suggest that an understanding of the genetic relation of correspondingly formed valleys may prove helpful elsewhere in the study of similar location problems.

2.—The engineer may obtain valuable data from published geological reports. These reports are generally of a scientific nature, but often contain information that can be used in a practical manner. They are related generally to the geology of a large area, and because of this may be helpful to the engineer in obtaining a broad perspective of the geology of a region, in some particular section of which an engineering project is proposed. The engineer must rely upon his own investigations for the geological information required within and adjacent to the construction lines of a structure.

3.—The data on excavation progress are important mainly in their manner of presentation. Because the geology of this project was followed so closely it was possible to draw an accurate geologic section covering the entire 24.6 miles of the tunnel. This fact, together with other related construction data and costs, correlates the geology with construction in considerable detail.

Acknowledgments

Some parts of this paper have been taken from reports to the Chief Engineer prepared jointly by Charles P. Berkey, M. Am. Soc. C. E., and the writer. Dr. Berkey was Consulting Geologist on the work. The writer also wishes to express his indebtedness to the Chief Engineer of the Metropolitan District Water Supply Commission, Frank E. Winsor, M. Am. Soc. C. E., who has shown a keen interest in the application of geology to engineering works of this kind.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

WATER SUPPLY ENGINEERING[1]

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION

The Committee reports to the Sanitary Engineering Division on its special field of activity—the noteworthy progress and trends in the science and art of water supply engineering for 1935. Generally speaking, the progress in any one year is relatively slight, and progress and trends tend to spread themselves for a series of years.

Current Construction

Some of the more important and notable water-works constructions for the past year, that may be mentioned herein, are the (200-mgd) filtration plant at Milwaukee, Wis., at an expected cost of $4 850 000; the $3 000 000 rehabilitation program at Cincinnati, Ohio, including increased filtering capacity, increased storage, and new mains; and the program at Denver, Colo., which contemplates the spending of $8 000 000 for the construction of a (11 000-acre-ft) storage reservoir, a (56-mgd) filtration plant, about 20 miles of pipe lines (up to 60 in. in diameter), and the lining of the west half of the Moffat Tunnel in order to bring in (about 50 000 acre-ft per yr) of west slope trans-mountain water. Attention is also called to the following projects:

The El Capitan Dam of the City of San Diego, Calif., and the appurtenant water-treatment plant costing approximately $3 000 000. The dam is approximately 220 ft high and is a combination rock and earth-fill structure. The design of the dam and the methods of construction were described[2] by the late Hiram Newton Savage, M. Am. Soc. C. E. The field-testing devices for the hydraulic fills have been reported[3] by E. A. Rowe, Assoc. M. Am. Soc. C. E.

The Colorado River Aqueduct, mentioned subsequently in more detail, and the Parker Dam are being constructed for the Metropolitan Water District of Southern Calnfornia. The latter is a dam of the arched type, located where foundation conditions necessitated a height of 320 ft, in order

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NOTE.—Written discussion on this Progress Report will be transmitted directly to the Chairman of the Committee for possible use in preparing subsequent reports.

[1] Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936.

[2] *Engineering News-Record*, July 13, 1933.

[3] *Loc cit.*, January 31, 1935.

to raise the water surface 85 ft. The length of the dam along the crest is 800 ft. A short description of this project has been published[4].

The Rodriguez Dam, in Mexico, was designed and constructed as a buttress type dam of record height, having a deep cut-off wall.

The Madden Reservoir and Dam was built to supplement the water supply of the Panama Canal[5].

The Chicago Avenue water supply tunnel, Dever intake crib, and Cermak pumping station, at Chicago, Ill., will cost approximately $15 250 000.

Tunnel No. 2 of the New York City water supply has been completed at a cost of $56 800 000.

The Quabbin Reservoir, and the Wachusett-Quabbin Aqueduct have been completed for the Metropolitan Water District of Massachusetts[6]. Approximately $65 000 000 was authorized to be expended on this work.

The manganese and iron removal plant at Lincoln, Nebr., costing about $211 000 shows modern trends in this division of water purification[7].

It would be well to examine in a more detailed manner the records of the Colorado River Aqueduct for the Metropolitan District of Southern California. To date, the work has been described, successively, by F. E. Weymouth[8], Julian Hinds[9], and J. L. Burkholder[10], Members, Am. Soc. C. E. The length of this aqueduct is about 240 miles. Records for tunneling have been made on this job, and new safety precautions have been taken in the use of explosives on the tunnel work. In this connection, special attention should be directed to the table of properties of aqueduct conduits by Mr. Hinds[11].

Fred C. Scobey, M. Am. Soc. C. E., has contributed an interesting paper entitled "Water-Conduits Construction in the West"[12], which deals with modern material and construction methods to increase flow capacity.

The building of large and interesting dams continues to show advance in methods of construction and novel methods of treating foundations; for instance, the Norris Dam of the Tennessee Valley Authority is known as a dam being built without a derrick, two cableways being used instead, each of which has a span of about 1 925 ft and lateral travel of 500 ft. The construction methods used at the Norris Dam have been described[13] by Ross White, M. Am. Soc. C. E., with particular reference to the cableways and the handling of aggregates. It was also described[14] in a paper entitled "Construction Methods at Norris Dam" by Verne L. Peugh, Assoc. M. Am. Soc. C. E., with particular reference to the handling of the concrete and of aggregates. It is also interesting to note the care and technique used in the foundation grouting for the dam, along with the unique devices developed to aid this work,—a "feeler" which determines the location, depth, and thickness of seams, and the "flusher", to wash and clean the openings before grouting. Records have been set for placing concrete at the Norris Dam, which have been accomplished through skillful mechanization of retail devices and general plan arrangements. The progress of placing concrete goes on with the improvement of pumps, air equipment, and vibratory tamping.

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[4] *Engineering News-Record*, November 29, 1934, p. 692.

[5] "Concrete for Madden Dam", by Irwin E. Burks, *Civil Engineering*, June, 1934, p. 288.

[6] "Boston's New Metropolitan Water Supply", by Frank E. Winsor, M. Am. Soc. C. E., *Civil Engineering*, June, 1934, p. 283; see, also, p. 295.

[7] *Engineering News-Record*, February 14, 1935.

[8] *Civil Engineering*, February, 1935, p. 72.

[9] *Loc. cit.*, p. 77.

[10] *Loc. cit.*, September, 1935, p. 524.

[11] *Loc. cit.*, February, 1935, Table 1, p. 80.

[12] *Loc. cit.*, September, 1935, p. 569.

[13] *Loc. cit.*, December, 1935, p. 737.

[14] *Loc. cit.*, p. 739.

Design problems have been simplified by the use of models, particularly in the case of the Bonneville Dam, in Oregon[15].

It may be well to note that the Boulder Dam closed its huge gates on February 1, 1936, and the normal flow of the Colorado River began to be stored at the rate of 5 000 cu ft per sec. This marks the completion of the largest and one of the most remarkable dams built to date. The Soil Erosion Service has announced its plans for maintaining a constant check on the quantity of sediment which will be deposited in the reservoir back of this dam during coming years.[16]

S. A. Thomas, Jr., Assoc. M. Am. Soc. C. E., has written a paper[17] dealing with the grouting procedure of the contraction joints in the Hogan Dam of the City of Stockton, Calif. This dam is of the arch type.

As mentioned in the 1934 Report[18] of the Committee there has been considerable progress in the principles of soil mechanics for the construction of rolled-fill dams. Charles H. Lee, M. Am. Soc. C. E., has outlined[19] a procedure for selecting material for rolled-fill dams, and has proposed graphical definitions for stability, water-tightness, and workability of materials.

Pipes and Pipe Lines

The past year has seen continued use of the established materials for pipes (cast-iron, steel, cement-asbestos, and concrete). Sectional Committee A-21 of the American Standards Association has done considerable work, which is nearing completion, on new specifications for cast-iron pipe, together with specifications for protective coatings and linings. Specifications for steel pipe and the standardization of gate-valves are progressing under the sponsorship of the American Water Works Association (Sub-Committees 7-A and 7-E, respectively). With the completion and publication of these new specifications, the water-works engineer will have at his disposal specifications representing the combined study and practice of engineers, consumers, and manufacturers.

Cement-asbestos and concrete pipe continue to be used as a substitute for cast iron and steel for pipe-line material. Large concrete pressure pipe has been installed and tested on the Colorado River Aqueduct up to 12 ft in diameter.

Linings

The effect of corrosive waters in reducing the carrying capacity of water mains has been investigated by a Committee of the New England Water

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[15] *Civil Engineering*, September, 1935, p. 604.

[16] *Engineering News-Record*, March 14, 1935.

[17] *Civil Engineering*, May, 1935, p. 296.

[18] *Proceedings*, Am. Soc. C. E., December, 1934, p. 1477.

[19] *Civil Engineering*, September, 1935, p. 556.

Works Association[20]. A marked difference was revealed between predicted and actual friction loss after a number of years of service. For tar-coated cast-iron pipe in service 30 yr, the average capacity loss when the water had a pH-value of 8.0 was approximately 30%; with a pH-value of 7.0, the loss was approximately 45%; and with a pH-value of 6.0, it was approximately 85 per cent. The report indicates that there is a considerable advantage in having a good lining or in having non-corrosive water. In the matter of lining, the 1934 Report of the Committee dealt with cement mortar lining for pipe lines[21] *in situ*, describing the work being done in England, New Zealand, and Australia.

PRIME MOVERS AND PUMPS

Diesel and gasoline engines continue to be more widely used as prime movers for pumps. The Internal Combustion Engine Institute has adopted, during the past year, rules for testing and rating combustion engines.

It may be of interest to mention the development of a high-speed plunger pump by Dr. Von K. Schoene, in Germany. This pump may be operated at a speed to allow it to be directly connected to a Diesel-engine drive without the use of reduction gearing.

WATER TREATMENT

The outstanding developments during 1935 are improved facilities for mixing and flocculation, accomplished by mechanical means and, in step with the advancement in mechanical equipment, there has been improvement in coagulants and zeolites. At Richmond, Va., the use of chlorinated-copperas has resulted in a better filtered water and a saving of approximately $10 000 per yr. Bulk-handling of chemicals continues to result in plant savings. Attention is called to the papers[22] of L. H. Enslow, Assoc. M. Am. Soc. C. E., on these subjects.

Until quite recently chemical feed-control devices have been volumetric. A distinct forward step has been the use of gravimetric feed in which the rate is controlled by weight through the operation of a synchronous motor-driven traveling weight on the scale beam.

Greater attention is being given to maintaining a clean condition of sand beds in filters by installations of various types of surface-washing equipment. The usual installation consists of perforated pipes suspended slightly above the sand bed when in filtering position. When the bed is being washed and the sand is expanded, the perforated pipes and the jets therefrom are beneath the surface of the sand and produce a scrubbing action. The City of Kenosha, Wis., has demonstrated the efficiency of this type of equipment.

The new purification plant at Milwaukee, Wis., exemplifies many of the foregoing advancements, and has been described[23] by L. R. Howson, M. Am. Soc. C. E.

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[20] *Journal*, New England Water Works Assoc., September, 1935.

[21] *Proceedings*, Am. Soc. C. E., December, 1934, p. 1478.

[22] *Water-Works and Sewerage*, July and December, 1935.

[23] *Loc. cit.*, December, 1935, p. 403.

Experiments with anthracite coal screenings as a filter medium are being continued. The City of Denver is installing several anthracite coal filters in its new plant. Anthracite coal has also been used in the Little Falls, N. J., filtration plant. Experiments indicate that some advantage may be obtained in the use of a combination of sand and anthracite screenings as a filter medium.

Taste and Odor.—The elimination of taste and odor continues to be an important development in the water-works field. With the improvement in the quality of water supplies, the public has become taste conscious and odor conscious and is demanding better tasting water now than at any previous time. More than four hundred municipal plants are now using activated carbon, largely in the powdered form. Many water-works plants are using ammonia and chlorine for the production of better water. Ray L. Derby, Assoc. M. Am. Soc. C. E., has described[24] chlorinating methods at Los Angeles, Calif.

RECORDS

Unusual flood conditions in Central New York State and drought conditions in the West have demonstrated the necessity for more complete run-off and rainfall records. S. T. Harding, M. Am. Soc. C. E., has given[25] interesting data concerning the trend in available water supplies in the Great Basin area, which resulted from his study of records covering a period of 300 yr.

At its Annual Convention, at Los Angeles, Calif., in July, 1935, the Society prepared a resolution urging that the new National Resources Board establish a distinct agency to continue the preparation of a long-range plan for the development and control of water resources[26]. This agency has been created and is known as the Water Resources Committee. Through this Committee civil engineers may look forward to the availability of more extensive records on water resources.

ACKNOWLEDGMENTS

It is the Committee's wish to acknowledge the assistance received from engineers not connected with its work in the compilation of this report.

Respectfully submitted,

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JOHN R. BAYLIS,
WILLIAM W. BRUSH,
LOUIS R. HOWSON,
JEPTHA A. WADE,
THOMAS H. WIGGIN, *Chairman*,

Committee of the Sanitary Engineering Division
on Water Supply Engineering.

January 16, 1936.

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[24] *Civil Engineering*, September, 1935, p. 558.

[25] "Changes in Lake Levels in the Great Basin Area", *Civil Engineering*, February, 1935, p. 87.

[26] *Proceedings*, Am. Soc. C. E., February, 1936, p. 203.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

STRUCTURAL ALLOY AND HEAT-TREATED STEELS[1]

PROGRESS REPORT OF SUB-COMMITTEE NO. 2, COMMITTEE ON STEEL, OF THE STRUCTURAL DIVISION

In examining the field assigned to it for consideration, the Sub-Committee on Structural Alloy and Heat-Treated Steel decided that its initial endeavors should be confined to a survey of past and present practice of the use of steels for structural purposes that possess higher strength than that normally associated with plain carbon steel of structural grade. This progress report, therefore, will summarize briefly the results of such a survey to date.

In carrying on this work, the Sub-Committee, through the generosity of a member of the Society, had placed at its disposal a small fund that enabled it to employ, for a time, Dr. J. M. Frankland, who made a quite thorough bibliographical search of engineering literature. This search was greatly facilitated by having placed at the disposal of the Sub-Committee, through the courtesy of Dr. G. B. Waterhouse and Mr. F. T. Sisco, respectively Chairman and Editor of the National Research Council's Iron Alloys Committee, the entire and comprehensive file of that Committee. Furthermore, the Sub-Committee has had the benefit of data supplied by a number of individuals and the more important steel companies. The Sub-Committee, for itself and the Society, expresses its thanks to those individuals and companies who have co-operated in its work.

In discussing high-strength steels (that is, steels other than plain carbon steels of structural grade), it will facilitate matters somewhat if they are classified according to the principal alloy, other than carbon, used in their composition, as follows: (1) Silicon; (2) manganese; (3) nickel; (4) chromium; (5) vanadium; (6) molybdenum; and (7) copper.

When incorporated in proper proportions with a plain carbon steel, each of these alloys, adds specific properties to the steel, some of which are desirable and some detrimental. For the latter reason, the metallurgist and steel maker attempt to save the good properties engendered by an alloy addition and overcome the detrimental ones by incorporating a second or third alloying element, keeping in mind always that the total cost of the additive alloys must be commensurate with the gains secured.

NOTE.—Discussion on this report will be closed in **August, 1936,** *Proceedings.*

[1] Presented at the meeting of the Structural Division, New York, N. Y., January 16, 1936.

It is to be noted that the Sub-Committee has used the expression "total cost." The increased cost of an alloy steel is not due to the additional cost of the alloy or alloys *per se*. The cost of the melting practice may be higher, there may be more cropping loss, the steel may roll harder and have to be rolled under better temperature control, and it may fabricate with greater difficulty. The last two will require more power, engender higher upkeep and maintenance costs, and may add extra operations, such as the necessity of drilling or sub-punching and reaming the thin sections or planing the sheared or flame-cut edges, to avoid injury in subsequent shop fabrication. There may also be more rejections due to surface imperfections, or failure to meet the specification requirements on account of the greater difficulty of making and treating an alloy steel. It will be recognized, therefore, that the demand and necessity for steel of higher strength entails complexity both from the alloying side and in handling.

For the purposes of this report the alloys will be discussed as listed, grouping together the last four (the chromium, vanadium, molybdenum, and copper steels) because at present they are more in the nature of experimental types with possibilities than the first three on the list, which have seen considerable use to date. In the discussion of the individual groups an effort will be made to note the changes toward the more complex alloy types.

Silicon Steel

The so-called silicon steel of structural engineering use has really been misnamed for, with few exceptions, it contains very little additive silicon as an alloy and, further, its properties are at least as dependent on another alloying element incorporated with it as they are on silicon. When adopted for construction use in the United States Navy it was accepted under the designation "H. T. S."; that is, high tensile steel, and it was in the fact that it possessed some enhanced properties over the previously used plain carbon steel that it had merit.

In steel-making silicon has two important uses: (*a*) As a deoxidizing agent in the making of a killed steel (and all plate and structural shapes are made from either acid Bessemer or basic open-hearth killed steels); and, (*b*) as a deliberate alloying element when it is incorporated in amount in excess of the quantity needed as a deoxidizer for steel.

As a deoxidizing element, a killed structural steel (which is ordinarily referred to as a plain carbon steel) has a silicon content from a trace to 0.02 per cent. As a deliberate alloying element, it may be used to produce: (1) An iron-carbon-silicon alloy known as "electrical sheet", low in carbon and having a silicon content of 0.5% to 4.5%; (2) a silicon-manganese spring steel with silicon content of 1.5% (which is not a structural steel, and is usually heat-treated); or (3) an iron-carbon-silicon steel containing also a substantial percentage of at least one other element, such as manganese. The so-called structural silicon steel of this report is of the latter type.

Naturally the term, "structural silicon", applies to an infinite variety of mixes, as carbon might range from 0.05% to 0.40%; silicon from 0.20% to

1.50%; and manganese from 0.50% to 1.60%; and the three could be combined so as to produce the physical properties associated with structural silicon. The term, "structural silicon", means, then, a low carbon to medium carbon steel, with enough silicon and manganese to raise the strength 30% to 40% above carbon structural steel without producing properties unsatisfactory for structural use; its formula is not more precise than that.

The tensile properties of structural silicon steel as specified in the United States are almost universally those of Specification A94, of the American Society for Testing Materials (A. S. T. M.), and can be most usefully considered by comparison with the ordinary A. S. T. M. bridge steel standard prior to 1933 (A7–29), and the same as that which was modified upward in 1933 (A7–33T, later A7–34) (see Table 1). The modulus of elasticity is not sensibly different for these three steels, varying between 29 000 000 and 30 000 000 lb per sq in. On the relationship between tensile properties and those under other kinds of stress not a great deal has been reported. Such experimental work as has been reported would indicate that the elastic limit in shear is about 60% of that in tension, the same relationship as exists for plain carbon steel of structural grade.

TABLE 1.—COMPARISON OF SPECIFICATIONS FOR STRUCTURAL SILICON STEEL

Description	SPECIFICATIONS, AMERICAN SOCIETY FOR TESTING MATERIALS:		
	A7 – 29	A7 – 34	A94 – 33
Tensile strength, in pounds per square inch	55 000 to 65 000	60 000 to 72 000	80 000 to 95 000
Yield point stress (drop of beam)	One-half of tensile strength	One-half of tensile strength	....
Yield point stress (minimum)	30 000	33 000	45 000
Percentage elongation in 8 in	$\frac{1\,500\,000}{\text{Tensile strength}}$	$\frac{1\,500\,000}{\text{Tensile strength}}$	$\frac{1\,500\,000}{\text{Tensile strength}}$

The first actual use of this type of steel seems to have been in the upper plates of S. S. *Mauretania*, built in 1907. The following is reported to have been their analysis and physical properties:

Carbon (percentage)	0.27
Silicon (percentage)	1.12
Manganese (percentage)	0.72
Tensile strength, in pounds per square inch	92 000 to 105 000
Yield point stress, in pounds per square inch	65 000
Elongation in 8 in. (percentage)	25 to 30

Steel of this type does not seem to have been used since in either foreign or domestic ship construction. Some use of silicon steel has been made in naval vessels but in the U. S. Navy this use has ceased. Captain A. H. Van Keuren, U. S. N., advises that under the Navy Specification for "High Tensile Steel", which did not specify the alloying chemistry, silicon steel was accepted and used with satisfaction for riveted work. He continues:

"The advent of welding, and its extension to important structural members of ships' hulls, has caused this Bureau to investigate the behavior of various high tensile steels after welding. As a result of this investigation, 'silicon steels' will no longer be allowable under the specification, for welding purposes, because the steel manufacturers were found to be unwilling to bid on the carbon and manganese limits which it was found advisable to impose on steel containing moderate amounts of silicon (0.20–0.30%) to avoid unacceptable air-hardening. They feared rejections either for failure to meet the prescribed physical characteristics, on the one hand, or for an excess of hardening elements, on the other. 'High silicon' steels were found too susceptible to air-hardening after welding to be further considered."

It appears evident from the foregoing that silicon steel, as referred to in this report, is of structural, and not of ship-building, interest.

As yet, there has been no decided trend toward the use of alloy steel in building construction. In this field, the weight of the steel frame is not a factor of disadvantage—cost alone generally controls. For the usual building elements, the extra cost of an alloy steel is not regained by savings in weight. In such construction alloy type steel has been used only where carbon steel would have produced rather excessive sections. A few crane runway girders of exceptional span have been designed in silicon steel when it was found impracticable to assemble enough material of carbon steel in their flanges. In a few buildings of office or store type (Cleveland Union Terminal Company's Tower Building, Cleveland, Ohio, Merchandise Mart, Chicago, Ill., Marshall Field Building, Chicago, etc.), the columns required in the lower stories would have exceeded the available H-sizes if designed in carbon steel, and were designed with silicon H-sections to avoid the more bulky built-up sections. A few limiting deep beams have been similarly designed in silicon. In the Merchandise Mart, 11 100 tons out of 51 500, and in the Marshall Field Building, 14 900 tons out of 21 000, were of silicon steel. In the Richmond Power Station built in Philadelphia, Pa., in 1926, about 8 000 tons in a total of 10 000 were silicon steel, the stated advantage being the saving of space and head-room through the use of smaller columns and shallower beams.

The use of silicon steel in bridge construction began with the Metropolis Bridge, at Metropolis, Ill., built in 1915–16. The major span of this bridge (720 ft) was, and is, the longest "simple" truss span on record, and justified the introduction of a new high-strength material. Prior to that date, recourse frequently had been had to $3\frac{1}{4}$% nickel steel (with a yield point of 50 000 or 55 000 lb per sq in.) in long-span bridge construction (Quebec, Queensborough, etc.), but the unit price of this alloy had prevented its introduction into any but exceptionally long spans. After the construction of the Metropolis Bridge, the use of silicon steel in major bridges was uninterrupted. From 1915 to 1930 the American Bridge Company reports more than 120 000 tons in a partial list comprising twenty important structures. The Bethlehem Steel and the McClintic-Marshall Companies report between 1926 and 1932 more than 90 000 tons in sixteen bridges. The sum of these two reports by no means covers the total tonnage of structural silicon steel during this period.

The foregoing structures do not necessarily have great span lengths. They range from the towers and trusses of the 3 500-ft George Washington, the 1 850-ft Ambassador, and the 1 750-ft Philadelphia-Camden Bridges, and the 1 100-ft cantilever spans of Carquinez Straits Bridge, down through many railroad spans of 300 to 400 ft, to a group of short vertical lift spans (Kennebec River, 242 ft; and Passaic River, Pennsylvania Railroad, 230 ft). Not included in the foregoing are bridges under construction to-day: 20 000 tons of silicon steel in the Golden Gate Bridge; 26 000 tons in the East, and 42 000 tons in the West, Crossings, San Francisco Bay; 31 000 tons in the river spans and approaches of the Mississippi River Crossing, at New Orleans, La.; and 15 000 tons in the Tri-Borough Bridge, New York City. In all the period from 1916 to date, it is probable that the Castleton (N. Y.) Bridge with 400-ft and 600-ft spans, is the only structure with trusses of this length that was constructed of carbon steel throughout.

The tensile test requirements (in pounds per square inch) for the Metropolis Bridge were:

Ultimate tensile stress	80 000 to 95 000
Minimum yield point stress	45 000

These stress requirements have remained unchanged to the present time. In the further physical requirements there have been slight differences. For the Metropolis Bridge, the minimum elongation was specified as 17% and the minimum reduction of area as 35%; the latter was not always quite attained.

The specifications for the towers of the Philadelphia-Camden Bridge (1922) called for: A percentage elongation in 8 in. of $\frac{1\,600\,000}{\text{Tensile strength}}$ (approximately from 17 to 20% with reductions for thickness); and a reduction of area of 35% (with reductions for thickness, a minimum of 24 per cent). These latter clauses were adopted for several later bridges. Several others have been constructed under A. S. T. M. Specification A94 requiring a percentage elongation in 8 in. of $\frac{1\,500\,000}{\text{Tensile strength}}$ (approximately 16 to 19 per cent.). The permissible reduction of area was not specified.

The history of the use of structural silicon steel on bridge construction shows: That specification requirements for ultimate tensile strength and yield point stress have remained unchanged; that there has been a slight lowering of the requirements for elongation; that the reduction of area, an important measure of the ductility of the metal, has been quite largely eliminated; and that the requirements for the cold bend test (not quoted in this report) have been made less severe.

With reference to chemical specifications structural silicon steel falls into two categories, those that specify upper limits for silicon and manganese and those that do not. The specifications of the Delaware River Bridge (1922) are of the first class:

Maximum Allowable Percentage of:	
Carbon	0.40
Manganese	1.00
Silicon	0.45
Minimum Allowable Percentage of:	
Silicon	0.20

These limits were written into the specifications of many later bridges, including Mt. Hope, Louisville (Ky.) Municipal, Kennebec River, Arthur Kill, New Orleans, La., and San Francisco-Oakland. For the towers of the George Washington (Hudson River) Bridge and of the Golden Gate Bridge, the foregoing carbon and silicon limits were retained, but no limit was placed on manganese.

In A. S. T. M. Specification A94, used for the Ambassador (Detroit River) Bridge, Albany-Rensselaer, Troy-Menands, and other bridges, the same maximum carbon (0.40%) is specified, and the same minimum silicon (0.20%); but no upper limit is placed on silicon, and no mention is made of manganese. Those who set the upper limit of 1.00% for manganese later waived it in the case of the New Orleans Bridge (1933). All specifications agree in limiting phosphorus to 0.06% (acid) and 0.04% (basic), and sulfur to 0.05 per cent.

MANGANESE STEEL

For some years a type of open-hearth steel has been made which contains from 1.00 to 2.00% of manganese and which is generally referred to as an "Intermediate Manganese Steel" to differentiate it from the high abrasive resistance steels of the Hadfield type containing approximately 14% manganese. This type of steel has been used for heat-treated parts for automobiles and for large motor shafts, and has been, and is, under trial for rails. It was not suggested for structural purposes in the "as-rolled" condition until 1929. In that year The Port of New York Authority in its specification for the superstructure of the Kill van Kull Bridge (later named the "Bayonne Bridge") carried a provision that the contractor could substitute another alloy steel of satisfactory quality for the specified 3.25% nickel steel specified, if it had the equivalent physical properties and was approved by the Chief Engineer.

The successful bidder on this structure proposed the use of a steel of the intermediate manganese grade. Experimental studies on trial heats seemed to warrant its adoption and its approval by the Chief Engineer of The Port of New York Authority marked the first use of this type of steel in structural engineering. The subsequent specifications of the Port Authority required the following physical properties and chemical analysis:

Carbon (percentage)	0.30 to 0.45
Manganese (percentage)	1.60 to 1.80
Silicon (percentage)	0.20 to 0.45
Ultimate tensile strength (minimum), in pounds per square inch	90 000
Yield point stress, in pounds per square inch	55 000
Elongation in 8 in. (minimum percentage)	$\frac{1\,500\,000}{\text{Ultimate tensile strength}}$
Reduction in area (minimum percentage)	30

The Bayonne Bridge contains the only large quantity of structural carbon-manganese steel with such high physical properties that, thus far, has been put into service.

It should be noted that the specifications for both silicon structural steel and the intermediate manganese steel, discussed so far, permit of wide chemical variations, the emphasis in both grades being placed on their meeting a specification for physical properties. Furthermore, both types are similar in chemical characteristics with regard to the alloying elements. For all purposes of discussion, therefore, they may be classed together, which fact is in evidence to-day in that the present trend is toward a middle ground in the carbon-silicon-manganese series and one may, therefore, more correctly speak of the two types as representing a "silico-manganese" structural steel. This trend toward a silico-manganese structural steel possibly may be best indicated by reference to Table 2.

TABLE 2.—TREND TOWARD THE CARBON-SILICON-MANGANESE TYPE OF STRUCTURAL STEEL

Type of steel	PERCENTAGES OF:			Remarks
	Carbon	Manganese	Silicon	
Plain carbon grade......	....	....	....	A. S. T. M. Specification A7 – 29
Silicon steel.............	0.27	0.72	1.12	S.S. *Mauretania* (1907)
Silico-manganese........	0.29 to 0.35	0.52 to 0.74	0.27 to 0.40	Metropolis Bridge (1916)
Silico-manganese........	0.40*	1.00*	0.20 to 0.45	Towers of the Delaware River Bridge
Silico-manganese........	0.40*	No limit	0.20 to 0.45	George Washington Bridge (1922)
Silico-manganese........	0.40*	Not specified	0.20†	A.S.T.M. Specification A94
Intermediate manganese.	0.30 to 0.45	1.60 to 1.80	0.20 to 0.45	Bayonne Bridge

* Maximum. † Minimum; no maximum specified.

The reported tests on the silico-manganese steels used in recent major bridges show that practically all the material falls between two ranges of composition, of which Table 3 presents some representative instances. Former high silicon (such as that used in the S. S. *Mauretania*) has disappeared, and between the two analyses represented in Table 3 there is agreement as to lowering silicon, but a disagreement of perhaps secondary importance as to the relative advantages of carbon and manganese.

TABLE 3.—RANGE OF COMPOSITION, SILICO-MANGANESE STEEL

Description	MUNICIPAL BRIDGE, LOUISVILLE, KY.			TOWERS OF THE GOLDEN GATE (CALIF.) BRIDGE, AND APPROACHES TO THE NEW ORLEANS (LA.) BRIDGE		
	Minimum	Maximum	Average	Minimum	Maximum	Average
Number of accepted heats.......	181			First 51		
Percentage of:						
Carbon....................	0.28	0.40	0.345	0.22	0.32	0.27
Manganese................	0.62	0.95	0.77	1.14	1.40	1.25
Silicon....................	0.20	0.36	0.27	0.20	0.30	0.25
Ultimate tensile strength, in pounds per square inch.......	80 200	96 400	86 600	80 350	95 000	88 000
Yield point stress, in pounds per square inch..................	45 400	53 900	49 700	47 500	69 100	55 800
Percentage, elongation in 8 in....	17.7	26.0	22.0	18.0	28.0	21.9
Percentage, reduction in area	32.5	52.5	43.8	35.2	59.9	48.1

It is common knowledge that carbon increases in strength with a proportionate loss of ductility. Silicon is a de-gasifier, deoxidizer, and scavenger;

it is used for that purpose in "killed" steel, such as rail steel, but increases "piping" and is not ordinarily otherwise used in carbon and structural steel. When added to structural steel in quantities greater than that required of a scavenger (resulting in a designed alloy) it increases the yield point and ultimate strength, and does not reduce the ductility and toughness as much as if the same strength was attained by increasing the carbon. Manganese is also a deoxidizer and scavenger and is used for this purpose in "unkilled" steels. It is present in the normal carbon structural steel to the extent of 0.40%, or more. It is stated, furthermore, that manganese raises the true elastic limit, or no-set strength, to a greater fraction of the yield point than silicon. Since this statement is based on a laboratory test which is not usually made, it requires further confirmation. If verified, it would be an important argument for manganese from the structural designer's standpoint.

The relative strengthening effect of silicon and manganese is not very different. In the carbon and manganese range usual in structural steels, 1 point of carbon has about the effect of 9½ points of silicon, or 11 points of manganese; this relationship is not exact.

TABLE 4.—COMPARISON OF STEEL PRODUCTS

Description	I	II	III
Carbon (C) (percentages)	0.24	0.16	0.23
Manganese (Mn) (percentages)	0.49	1.00	1.40
Silicon (Sil) (percentages)	1.44	0.90	0.20
$C + \frac{Mn}{11} + \frac{Sil}{9\frac{1}{2}}$ (total carbon equivalent)	0.44	0.35	0.38
Tensile strength, in pounds per square inch	88 000	80 200	83 000
Yield point stress, in pounds per square inch	58 000	55 500	51 700
Elongation in 8 in. (percentage)	25	23.5	22
Reduction of area (percentage)	50.5	......	49
Izod impact, in foot-pounds	30.5	......	......

A comparison of three steels of very different composition, all giving tensile properties "as-rolled" within the desired range, is offered in Table 4, from which it is evident that considerations other than that of obtaining a specified strength and ductility will control the choice of composition.

NICKEL STEEL

The first important reference to nickel steel for structural purposes was the study[2] by J. A. L. Waddell, M. Am. Soc. C. E., which was begun in 1906. Its first application occurred in 1908 on the Blackwell Island, or Queensborough, Bridge, New York City, for which approximately 6 000 tons of eye-bars and eye-bar pins were fabricated. The material for this bridge was specified to have a minimum of 3.25% nickel with the following physical properties:

Minimum elastic limit, in pounds per square inch	48 000
Minimum ultimate tensile strength, in pounds per square inch	85 000
Elongation in 8 in. (percentage)	$\frac{1\,600\,000}{\text{Ultimate tensile strength}}$

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[2] "Nickel Steel for Bridges", by J. A. L. Waddell, *Transactions*, Am. Soc. C. E., Vol. LXIII (1909), p. 101.

The average characteristics for the material actually furnished were as follows:

Elastic limit, in pounds per square inch..............	55 000
Ultimate tensile strength, in pounds per square inch....	95 000
Reduction of area (percentage).....................	40
Elongation in 8 in. (percentage)....................	23
Content of (Percentage):	
Carbon	0.37
Manganese	0.75
Nickel	3.30

Since its first application on the Queensborough Bridge nickel steel has been furnished with about the same chemical analysis and has been designated as a $3\frac{1}{4}\%$ nickel, structural steel. The standard specification for this type of steel has been similar to that recommended by the A. S. T. M., which calls for not more than 0.45% carbon, 0.70% manganese, and nickel not less than 3.25%, with physical properties specified as indicated in Table 5.

TABLE 5.—STANDARD SPECIFICATIONS FOR $3\frac{1}{4}$ PER CENT. NICKEL STRUCTURAL STEEL

Description	Plates, shapes, and bars	Eye-bar flats unannealed	Eye-bar flats annealed
Ultimate tensile stress, in pounds per square inch.....................	85 000 to 100 000	95 000 to 110 000	90 000 to 105 000
Yield point stress, in pounds per square inch.........................	50 000	55 000	52 000
Elongation in 8 in. (percentage)......	$\frac{1\ 500\ 000}{\text{Ultimate tensile stress}}$	$\frac{1\ 500\ 000}{\text{Ultimate tensile stress}}$	20
Elongation in 2 in. (percentage)......	18	16	20
Reduction of area (minimum percentage)...........................	25	25	35

The tonnage and types of material in the more important bridges, in which the $3\frac{1}{4}\%$ grade of nickel steel has been used, are presented in Table 6.

TABLE 6.—USE OF $3\frac{1}{4}$ PER CENT. NICKEL STEEL

Bridge	Type	Date completed	Tonnage	Application
Queensborough, N. Y. City	Cantilever....	1908	6 000	Eye-bars and eye-bar pins
Manhattan, N. Y. City....	Suspension....	1909	8 000	Stiffening trusses
Kansas City, Mo..........	Vertical lift...	1912	300	Lift span
St. Louis (Mo.) Municipal..	....	1916	8 000	Plates, shapes, and eye-bars
New Quebec..............	Cantilever....	1917	16 000	Stiffening trusses and eye-bars
Philadelphia-Camden......	Suspension....	1926	18 700	Stiffening trusses
San Francisco-Oakland Bay.	Cantilever....	*	3 700	....

* Not completed to date (1936).

Although this type of steel has found favor among engineers as indicated, there has been a tendency in recent years to displace it in plates and shapes with the silico-manganese types, and in eye-bars with heat-treated carbon steel. The reasons for this trend seem to lie in the difficulties experienced in rolling and shop fabrication and the high cost of the alloying element, nickel.

Because competitive prices seemed to be the controlling factor that governed its use, an attempt has been made recently to develop a lower priced nickel steel having approximately the same physical properties as the heretofore standard 3¼% grade. This newer grade (which may tentatively be designated as 1¾% to 2¼% nickel structural steel) is in reality a nickel-manganese steel, because the reduction in the nickel content has been compensated for by an increase in manganese. This steel was experimented with, and offered to the Port of New York Authority for use in the Bayonne Bridge.

Table 7 indicates the properties possible of attainment with this type of steel in the "as-rolled" condition. These results indicate that the 1¾% to 2¼% grade of nickel steel with higher manganese content will probably meet the specification requirements of the heretofore standard 3¼% nickel grade, although to date no data are available as to its properties as eye-bars, flats, etc.

TABLE 7.—PROPERTIES OF NICKEL-MANGANESE STEEL, AS ROLLED
(Composition: Carbon, 0.33%; Manganese, 0.93%; Phosphorus, 0.022%; Silicon, 0.024%; and, Nickel, 2.23%.)

Rolled section	Yield point, stress, in pounds per square inch	Ultimate tensile stress, in pounds per square inch	Elongation in 8 in. (percentage)	Reduction in area (percentage)	Bend test
1-in. sheared plate	62 700	99 600	17.7	32.9	"O.K." longitudinally
1-in. sheared plate	63 460	99 300	17.7	45.1	"O.K." longitudinally
1-in. sheared plate	62 400	99 440	18.7	43.2	"O.K." transversely
1-in. sheared plate	64 700	99 140	18.0	32.4	"O.K." transversely
1½-in. sheared plate	63 900	99 420	16.0	25.1	"O.K." longitudinally
1½-in. sheared plate	58 440	99 620	17.0	28.2	"O.K." longitudinally
1½-in. sheared plate	62 060	99 560	16.2	25.1	"O.K." transversely.
1½-in. sheared plate	61 080	101 500	15.0	23.3	"O.K." transversely
8 in. by 8 in. by 1½-in. angle	58 740	98 300	21.5	43.9	"O.K." longitudinally
15-in., 75-lb., I-beam	62 900	102 100	18.5	44.2	"O.K." longitudinally

Quite recently a proposed change in the A. S. T. M. Specification for nickel steel has been promulgated, which specifies the following chemical analysis:

Carbon (maximum, percentage)	0.45
Manganese (maximum, percentage)	0.85
Phosphorus (maximum, percentage)	0.05
Sulfur (maximum, percentage)	0.063
Nickel (percentage)	3.00 to 4.00
Copper, when copper steel is specified (minimum, percentage)	0.20

Material made according to this analysis is expected to develop:

Tensile strength, in pounds per square inch	90 000 to 115 000
Minimum yield point, in pounds per square inch	0.5 tensile strength
—but in no case less than	55 000
Elongation in 8 in. (minimum, percentage)	$\frac{1\,600\,000}{\text{Tensile strength}}$
Elongation in 2 in. (minimum, percentage)	$\frac{1\,700\,000}{\text{Tensile strength}}$
Reduction in area (minimum, percentage)	30

and full-sized tests of annealed eye-bars are expected to conform to the following requirements:

Tensile strength, in pounds per square inch....	85 000 to 100 000
Yield point (minimum), in pounds per square inch	48 000
Elongation in 18 ft (minimum, percentage)...	10
Reduction in area (minimum, percentage).....	30

It should be recalled that the use of nickel as an alloy was about the earliest attempt to furnish a structural steel with strengths higher than the plain carbon steels of structural grade. It was added to give increased toughness with increase in strength. As an alloying element it is unique for the purpose, as it is one of the few alloys available that definitely goes into solution with the weaker element of steel, the iron or ferrite. Most of the other alloying elements combine with the cementite, thus augmenting the carbide or hardening elements of steel. Furthermore, the modulus of elasticity of nickel is greater than that of steel so that its incorporation as an alloy tends to maintain the higher modulus value of steel. For these reasons nickel should have its place in a structural steel, provided the enhanced properties it imparts are commensurate with its increase in cost.

The foregoing résumé of the history of structural nickel steel indicates that economic conditions are forcing a reconsideration of its position in structural engineering so that, at present, as with the silico-manganese-group, it may be considered as being in an experimental stage.

Chromium, Vanadium, Molybdenum, and Copper Steels

The final group of steels for consideration are those containing chromium, vanadium, molybdenum, and copper. When these elements are used as alloys they are either used singularly with a plain carbon steel, in combination with each other, or in combination with silicon, manganese, or nickel. In most part, practically all steels of this group are distinctly in the experimental stage as structural engineering steels. Each of these elements adds its definite effect to a plain carbon steel, and for that reason, these alloys are used most often in combination, either to enhance each other's additive properties or to give properties not characteristic of any of them when used alone.

Chromium, when added to a carbon steel, gives increased strength, surface hardness, and depth of hardening, with some loss of ductility. It has no scavenger effect in cleaning a steel during the steel-making process. It forms complex carbides with the iron and carbon of a steel. As alloys that increase carbide formation act essentially as a higher carbon steel, they, therefore, become increasingly more air-hardened. For this reason chromium is generally used with a second alloy to mitigate its hardening characteristics.

Vanadium will effect a certain degree of deoxidization if the opportunity is afforded, but it is not an economical agent for this purpose and it is not as powerful a deoxidizing element as those commonly used. It is for its alloying effect that it is utilized. As an alloy it restricts grain size and growth and affects the properties of the ferrite and carbides of the steel, a small amount probably going into solution in the ferrite and the larger proportion forming complex carbides. Therefore, it tends to increase strength without loss in ductility, but more particularly it tends to increase the elastic limit, yield point, and impact resistance. Its value is enhanced when used with a second alloying element.

Molybdenum is not a scavenger nor a deoxidizer. It acts wholly as an alloy. Its value seems to lie in the fact that it acts in large measure like vanadium, thereby adding toughness and ductility as well as strength. As an alloying element it also seems to produce its best results when incorporated with a second alloy. With such additive alloys it increases the elastic properties when used in quantities less than 0.50% in the hot-rolled or normalized condition. It also increases the free-scaling characteristics of a steel which should make it valuable from the rolling-mill standpoint.

Copper is the third of the alloying elements that goes into solution in the iron or ferrite of steel and, therefore, produces rather marked effects in physical properties, particularly the yield stress. It appears that the first attempt to use copper in steel occurred more than 100 yr ago, and for a long period subsequently it was condemned as productive of red-shortness and classed in this respect with sulfur. About the turn of the Twentieth Century it was disclosed that copper by itself, up to a 4% content, did not make steel red-short, and within the last decade the reasons for these apparent contradictions have been explained. Low-carbon steel with a copper content up to 2% will not be a red-short if hot-worked between 1 500° and 1 800° F, but if hot working temperatures reach 2 000° F, a fissure brittleness develops in the steel.

In the normal heating of a copper-bearing steel the iron is oxidized more readily than the copper and there results a dispersion of pure copper within the scale. As the temperature is increased to the melting point of copper, 1 980° F, the copper in liquid form penetrates between the grain boundaries of the steel, causing the embrittlement first recorded by observers as "hot-shortness." Recent investigation has shown that the incorporation with copper of a small percentage of another element (chromium or nickel, for example), which at the higher temperatures produces mixed crystals the fusion point of which is higher than the temperature of hot working, will overcome the apparent red-shortness at the higher working temperature.

The apparent baneful effects of copper resulted in its use being limited for some time to the so-called copper-bearing steels with copper content of 0.20% in recognition of its effect in reducing corrosion; and it was not until recently, in studying its effect on corrosion resistance that it was recognized that it also had decided effects upon the mechanical properties. As a result of the recognition of this fact, a second class of copper steels, interesting from the structural engineer's standpoint, has begun to appear. This alloy contains more than the normal 0.20% copper of the so-called copper-bearing type for corrosion resistance. Associated with the copper in most instances in this newer class is a higher manganese content or an addition of nickel, chromium, or molybdenum.

The second class of copper-bearing steels has primarily been developed abroad. Investigational work is in progress in America, however, and announcements of steels of this type, may be expected soon.

There are other elements that will alloy with steel or may be used as scavengers, but for the most part, with a few exceptions, they need no consideration at this time either because their effects are not well understood or because their use for structural steels would not be warranted on account of their scarcity and high cost. The exceptions are titanium and aluminium and they should be considered because of their quite prevalent use to-day in steel-making.

Titanium is a deoxidizer and scavenger. Used after a first deoxidization with silicon and manganese, it seems to act as a "clean-up" deoxidizer, and its oxide acting in turn as a flux on the other oxides in the bath decreases their viscosity and, hence, facilitates their removal from the bath. Practically none of the titanium remains in the steel. In this respect it differs from aluminum with oxides that seem to have no fluxing action and remain in the steel in most part as a refractory particle. Titanium, therefore, is a scavenger and deoxidizer that tends to promote cleanliness in steel. It has no effect on grain size and not being retained in the steel can affect the properties of a steel only through its cleansing action.

Aluminum is a scavenger used primarily after silicon and manganese additions, and is the agent for killing or completely deoxidizing the ordinary grades of low-carbon structural steel. Used as a scavenger it occurs in steel, therefore, not as an alloy, but in the form of minute particles of its refractory oxide. Until quite recently its scavenging properties were considered its main merit. It is now recognized that when used in quantities in excess of that required for "killing" a steel, it has a decided effect upon the intrinsic grain size of the steel, this effect being due to the uniform dispersion of the multitudinous small particles of its oxide acting as nuclei for crystallization within the ingot mould and possibly also to some solution of the aluminium in the ferrite. Its use in the control of intrinsic grain size is of quite recent development and, as yet, not thoroughly understood.

It will be realized that in the foregoing group of alloying elements, especially when combined with silicon, manganese, and nickel there are numerous possibilities of combination. A great variety of them have already been subjected to experiment, developed, and put to use, for it is as heat-treated steels that the alloys show to greatest advantage, and it is only by heat-treatment that the full value of the alloying element can be secured as full compensation for their increased cost.

Until quite recently there has been no development of steels with this group of alloys that could be classified as of structural grade. The need for steels capable of withstanding higher stresses for tank work, etc., for greater corrosion resistance, for lighter weight mechanisms, and greater weldability, as well as the urge to increase sales of particular alloys, has resulted in recent years in increased consideration of the possible field for such alloys in the structural grades; that is, those which may be used in the "as-rolled" condition, or with simple and not too exacting heat treatments, such as stress relieving or normalizing. The development of this latter group has been so recent that with perhaps a few exceptions it must be considered as still in the experimental field. The economic status of this group, especially for structures, is therefore undetermined.

The great number of possibilities incident to the group of alloys now being reviewed, makes it undesirable to burden this report with the full data that have been assembled by this Sub-Committee. Therefore, this report is confined to steels that may be considered representative of some possible combinations within the grouping, selecting those for presentation that have indications of merit as structural steels.

One of the earliest of this group in time of use was Mayari steel, a chrome-nickel combination that was incorporated in the Harahan Bridge, a double-track, cantilever, railroad bridge over the Mississippi River at Memphis, Tenn., and built during 1914 and 1915. Of approximately 16 000 tons of steel in this structure, 8 700 tons were of Mayari steel, 6 950 tons of the latter in riveted members and 1 750 tons in eye-bars.

Mayari steel gets its name from the locale of its origin, a Cuban deposit of ore rich in nickel and chromium. Mayari pig iron contains from 0.80% to 1.25% nickel and 1.60% to 2.50% chromium. In the open-hearth this pig is charged in only sufficient quantity to give a total chromium content of less than 0.60% and a nickel addition is made sufficient to produce a steel that will meet the requirements for 3¼% nickel structural steel. The following tabulation gives the physical properties of the Mayari steel used in the Harahan Bridge:

Average ultimate tensile strength, in pounds per square inch..	95 000
Average elastic limit, in pounds per square inch.............	56 000
Average elongation in 8 in. (percentage)....................	16

There is still a ready demand for Mayari pig iron as a cupola addition for the improvement of automotive and other iron castings, and it has also been used for cast steel. Its use as a structural steel has ceased, due entirely to economic conditions.

In an effort to produce a steel of higher strength than plain carbon steel, and, at the same time, maintain high ductility and good cold-working qualities in the "as-rolled" condition, there has been developed and used quite widely a steel that goes under the trade name of "Cromansil." Its name is indicative of its alloying elements. In general analysis it is of the following type:

	Percentage
Carbon	0.15 to 0.25
Manganese	1.10 to 1.40
Silicon	0.60 to 0.90
Chromium	0.40 to 0.60

Similar to many other steels Cromansil probably should be looked upon as a class of steels because, as furnished, its analysis is varied over quite wide ranges, depending on the physical properties desired or the thickness of material to be furnished, as is exhibited by the data in Table 8. It is

TABLE 8.—PHYSICAL PROPERTIES OF TYPICAL "CROMANSIL" STEELS
("As-Rolled" or Normalized)

Type of material	ANALYSIS (PERCENTAGE)				Condition	Yield point stress, in thousands of pounds per square inch	Ultimate tensile stress, in thousands of pounds per square inch	Elongation (percentage in 8-in.)	Reduction in area (percentage)	Izod impact, in foot-pounds	Brinell hardness number
	Carbon	Chromium	Manganese	Silicon							
1-in., round stay-bolts	0.05	0.25	1.12	0.87	As-rolled.....	54	71	31	75	64	125 to 135
Boiler-Plate:											
½-In. Thick:											
Longitudinal....	0.21	0.47	1.17	0.72	As-rolled.....	50	92	20	56	36	200 to 220
Transverse......					As-rolled.....	51	91	18	54	..	....
1-In. Thick:											
Longitudinal....					As-rolled.....	51	99	16	..	..	200 to 220
Transverse......					As-rolled.....	51	98	14	..	25	....
1½-In. Thick:											
Longitudinal....					As-rolled.....	56	89	19	..	..	200 to 220
Transverse......					As-rolled.....	53	89	17	..	24	....
2-In. Thick:											
Longitudinal....					As-rolled.....	53	86	20	..	..	200 to 220
Transverse......					As-rolled.....	54	88	15	..	20	....
2-In. Thick.......	0.21	0.51	1.09	0.064	Normalized...	53	94	20	..	..	220 to 220
⅜-In. Thick.......	0.22	0.43	1.15	0.87	Normalized...	74	105	29*	50	32	210 to 230
Structural Shapes:											
2 by 2-In. Bar:											
Center..........	0.32	0.46	1.07	0.44	As-rolled.....	65	99	27*	64	..	200 to 220
Edge...........					As-rolled.....	68	107	25*	62	..	....
⅜-In. Plate	0.31	0.42	1.13	0.87	As-rolled.....	72	119	23*	..	42	225 to 245
⅜-In. Plate:											
Longitudinal....	0.46	0.67	1.19	0.82	As-rolled.....	88	142	20*	..	..	265 to 285
Transverse......					As-rolled.....	91	143	19*	..	20	....
Longitudinal....					Normalized...	100	148	20*	..	..	....
Transverse......					Normalized...	101	140	20*	..	23	....

* Results marked * are percentages in 2 in.

TABLE 9.—PHYSICAL PROPERTIES, VANADIUM AND MOLYBDENUM
(All Tests Are on Material in the "As-Rolled" Condition)

Test No.	TENSION TESTS		Yield point stress, in pounds per square inch	Ultimate tensile stress, in pounds per square inch	PERCENTAGE ELONGATION:		Percentage reduction of area	Brinell hardness number	BEND TEST	
	Direction	Plate thickness, in inches			In 2 in.	In 8 in.			Degrees of bending before failure	Remarks
(*a*) CHROME VANADIUM PLATE (ANALYSIS: CARBON, 0.17%; MANGANESE, 0.47%; SILICON, 0.25%; CHROMIUM, 0.92%; AND VANADIUM, 0.19%)										
45L– 1	Longitudinally	1½	59 500	85 600	46.0	19.5	52.3	....	180	No failure
45L– 2	Longitudinally	1½	60 200	85 600	43.5	19.5	50.1	....		
45L– 3	Transversely	1½	60 450	85 850	48.0	21.6	52.4	....	180	No failure
45L– 4	Transversely	1½	61 200	87 250	47.5	20.6	51.5	....		
45L–25	Longitudinally	¾	55 250	82 900	42.0	17.5	55.9	....	180	No failure
45L–26	Longitudinally	¾	54 550	82 350	42.0	17.7	54.4	....		
45L–27	Transversely	¾	54 350	84 350	37.5	17.2	54.4	....	180	No failure
45L–28	Transversely	¾	56 650	84 900	39.0	18.4	56.2	....		
45L–49	Longitudinally	⅜	57 200	77 900	39.5	20.6	57.1	....	180	No failure
45L–50	Longitudinally	⅜	53 050	77 150	42.5	21.5	59.4	....		
45L–51	Transversely	⅜	53 500	75 650	37.5	19.2	57.5	....	180	No failure
45L–52	Transversely	⅜	55 000	76 850	45.0	23.5	55.5	....		
(*b*) CHROME VANADIUM PLATE (ANALYSIS: CARBON, 0.25%; MANGANESE, 0.75%; SILICON, 0.20%; CHROMIUM, 1.00%; AND VANADIUM, 0.20%)										
45H– 1	Longitudinally	1½	60 700	104 050	47.0	19.1	53.6	....	180	No failure
45H– 2	Longitudinally	1½	70 450	104 300	44.0	18.6	52.0	....		
45H– 3	Transversely	1½	73 150	103 750	40.5	17.6	43.3	....	180	No failure
45H– 4	Transversely	1½	70 800	106 600	35.0	15.5	43.0	....		
45H–25	Longitudinally	¾	89 300	123 650	26.0	11.1	38.1	....	102	....
45H–26	Longitudinally	¾	89 550	123 650	29.5	12.4	40.1	....		
45H–27	Transversely	¾	90 700	129 600	26.5	12.4	34.4	....	47	....
45H–28	Transversely	¾	89 600	126 850	27.0	13.4	37.1	....		
45H–49	Longitudinally	⅜	97 100	124 100	24.5	11.5	43.4	....	180	Slight opening*
45H–50	Longitudinally	⅜	95 950	123 500	26.0	11.2	41.4	....		
45H–51	Transversely	⅜	102 350	126 000	24.0	11.5	39.1	....	87	Slight opening*
45H–52	Transversely	⅜	101 700	126 750	20.5	11.4	33.6	....		
(*c*) MANGANESE VANADIUM PLATE (ANALYSIS: CARBON, 0.14%; MANGANESE, 1.42%; SILICON, 0.20%; AND VANADIUM, 0.18%)										
45A– 1	Longitudinally	1½	68 100	93 400	40.0	17.5	44.0	....	180	No failure
45A– 2	Longitudinally	1½	69 350	93 600	43.0	18.7	46.3	....		
45A– 3	Transversely	1½	66 600	90 350	42.0	18.5	46.2	....	180	No failure
45A– 4	Transversely	1½	67 850	91 900	30.0	16.2	31.6	....		
45A–25	Longitudinally	¾	74 250	97 650	41.0	18.5	56.6	....	180	No failure
45A–26	Longitudinally	¾	74 050	98 150	40.0	18.6	55.8	....		
45A–27	Transversely	¾	74 550	97 550	38.0	17.6	51.0	....	180	No failure
45A–28	Transversely	¾	74 200	97 800	38.5	18.1	52.6	....		
45A–49	Longitudinally	⅜	75 500	98 400	34.0	17.1	53.2	....	180	No failure
45A–50	Longitudinally	⅜	74 900	98 400	34.0	16.9	52.2	....		
45A–51	Transversely	⅜	75 850	97 900	35.5	17.2	54.2	....	180	Slight opening*
45A–52	Transversely	⅜	76 250	98 350	34.0	17.5	54.5	....		
(*d*) NICKEL VANADIUM PLATE (ANALYSIS: CARBON, 0.21%; MANGANESE, 0.91%; SILICON, 0.30%; NICKEL 1.21%; AND VANADIUM, 13.0%)										
45.4	Longitudinally	1½	74 550	99 500	42.5	20.5	45.4	....	165	....
45.5	Longitudinally	1½	71 150	98 600	33.0	16.3	35.8	....		
45.1	Transversely	1½	75 500	101 900	24.0	14.1	23.0	....	70	....
45.2	Transversely	1½	73 300	100 100	24.5	14.1	24.0	....		
45.22	Longitudinally	¾	78 200	105 000	34.5	15.3	45.2	....	180	No failure
45.24	Longitudinally	¾	73 050	100 350	38.5	17.7	51.8	....		
45.19	Transversely	¾	74 400	100 850	29.0	14.7	37.8	....	180	Broke†
45.21	Transversely	¾	75 000	102 500	31.0	16.7	37.2	....		
45.45	Longitudinally	⅜	79 150	100 550	29.0	13.7	47.1	....	180	Slight opening‡
45.47	Longitudinally	⅜	78 500	100 350	23.5	11.2	44.5	....		
45.42	Transversely	⅜	78 700	99 100	21.0	10.6	35.5	....	180	Slight opening‡
45.44	Transversely	⅜	80 000	98 700	24.0	11.8	38.0	....		

* Slight opening after pinching between parallel blocks.
† Broke when pinched between parallel blocks.
‡ Opened after considerable pinching between parallel blocks.

TABLE 9.—(*Continued*)

Test No.	TENSION TESTS		Yield point stress, in pounds per square inch	Ultimate tensile stress, in pounds per square inch	PERCENTAGE ELONGATION:		Percentage reduction of area	Brinell hardness number	BEND TEST	
	Direction	Plate thickness, in inches			In 2 in.	In 8 in.			Degrees of bending before failure	Remarks
(*e*) CARBON MOLYBDENUM PLATE (ANALYSIS: CARBON, 0.245%; MANGANESE, 0.85%; SILICON, 0.19%; PHOSPHOROUS, 0.020%; SULFUR, 0.037%; MOLYBDENUM, 0.22%; AND COPPER, A TRACE)										
......	....	1½	56 400	79 900	42.5	21.5	....	194	....	Plate specimen
......	....	⅝	60 450	88 200	29.5	14.7	....	194	....	Plate specimen
......	....	½	61 600	85 300	23.7	19.3	....	192	....	Plate specimen
......	....	⅜	65 660	87 000	34.7	19.9	....	179	....	Plate specimen
(*f*) CARBON MOLYBDENUM PLATE (ANALYSIS: CARBON, 0.26%; MANGANESE, 0.92%; SILICON, 0.13%; PHOSPHORUS, 0.018%; SULFUR, 0.026%; AND MOLYBDENUM, 0.29%)										
......	....	¼	64 600	92 450	31.0	16.1	....	196	....	Plate specimen
......	....	⅝§	60 000	91 000	24.0	....	55.5	187	....	Plate specimen
(*g*) COPPER MOLYBDENUM PLATE (ANALYSIS: CARBON, 0.30%; MANGANESE, 0.77%; SILICON, 0.26%; PHOSPHORUS, 0.24%; SULFUR, 0.027%; MOLYBDENUM, 0.20%; AND COPPER, 0.24%)										
......	....	¾	56 190	89 400	....	20.4	42.5	....	180	No failure
......	....	½	56 000	90 560	....	21.7	49.9	....	180	No failure
(*h*) COPPER MOLYBDENUM PLATE (ANALYSIS: CARBON, 0.25%; MANGANESE, 0.86%; SILICON, 0.20%; PHOSPHORUS, 0.30%; SULFUR, 0.019%; MOLYBDENUM, 0.21%; AND COPPER, 0.55%)										
......	....	¼	67 500	91 550	31.0	17.5	....	196	....	Plate specimen
......	....	⅝§	59 000	91 000	....	20.5	46.6	179	....	Plate specimen

§0.505 in.

claimed that steels of this class, "cold work" and fabricate readily, will not harden when "flame cut", and are readily welded.

To date more than 100 000 tons of Cromansil have been used in tubing for oil wells, approximately 500 tons have been rolled into structural shapes, and several thousand tons have been rolled into plates and sheets.

Vanadium has been used for some time principally in combination with chromium or nickel and, in the heat-treated condition, for automotive and other dynamically stressed parts. One of the recent developments of promise, in the use of vanadium, has been the formulation of an analysis containing low carbon, relatively normal silicon, and a high manganese content in connection with the usual vanadium addition. Table 9(*a*), 9(*b*), 9(*c*), and 9(*d*) indicate the effect of vanadium when incorporated with other alloying elements.

Molybdenum likewise has been used principally as an alloy in heat-treated parts for dynamic stressing. Since this Sub-Committee began its work, the possibilities of this alloying element for a high-strength structural steel has been initiated in an entirely experimental way with promising results. In this case, again, however, as with vanadium, the molybdenum in normal quantities has been incorporated in relatively low-carbon steels of normal silicon content and increased manganese content. Following recent trends in European practice it has been further suggested that a small quantity of copper might be incorporated as an additive protection against corrosion. Data on these experimental heats of molybdenum-manganese steels are given in Table 9(*e*), 9(*f*), 9(*g*), and 9(*h*).

Recently, several types of steel containing copper have been proposed. One of these steels (Cor-ten) is a chrome-silicon type that is high in phosphorus, and the other (Man-ten) is a silico-manganese type. Their composition and physical properties in the as-rolled condition are as follows:

Cor-ten Steel:

Carbon (maximum percentage)	0.10
Manganese (percentage)	0.10 to 0.30
Phosphorus (percentage)	0.10 to 0.20
Sulfur (maximum percentage)	0.05
Silicon (percentage)	0.50 to 1.00
Chromium (percentage)	0.50 to 1.50
Copper (percentage)	0.30 to 0.50
Ultimate tensile strength, in pounds per square inch	70 000
Yield point stress, (minimum) in pounds per square inch	50 000
Percentage elongation in 8 in.	20

Man-ten Steel:

Carbon (percentage)	0.20 to 0.35
Manganese (percentage)	1.25 to 1.70
Phosphorus (maximum percentage)	0.04
Sulfur (maximum percentage)	0.05
Silicon (minimum percentage)	0.15
Copper (minimum percentage)	0.20
Ultimate tensile strength, in pounds per square inch	85 000
Yield point stress (minimum) in pounds per square inch	55 000
Percentage elongation in 8 in.	18

Both these steels are of the air-hardening type and require considerable care in their manipulation in the manufacturing processes; as yet, they have not been used to any considerable extent for structural purposes.

Up to this point the report has been confined to a necessarily short survey of the development and present status of American high-strength structural steels. This study would not be complete, however, without some reference to similar endeavors in the European countries.

Foreign engineering practice still endorses the use, to a great extent, of steels very similar to Bessemer and open-hearth ordinary carbon structural steel containing 0.15% to 0.20% carbon. Outside of Germany, Austria, and the United States, there has been little use of any other steels. In Great Britain there had been some interest in high-strength ship plates before the World War, the steamships, *Lusitania* and *Mauretania*, being plated in the midship portions with silicon steel. When the Washington Naval Treaty was signed, in 1922 a new incentive was supplied for the use of high-strength steel in ship-building. The British have concentrated on the development of a steel with a high elastic limit (as distinct from yield point), the plate to be so cooled from the rolling as to be practically normalized. Considerable research has been conducted at Woolwich Arsenal on manganese, silicon-manganese, nickel, and chromium-copper steels.

The study on the effect of manganese resulted in the suggestion of a normalized (air-cooled from 1 560°–1 500°F) plate material containing 0.25% to 0.35% carbon, and 1.20% to 1.70% manganese. A typical example of this material is as follows:

Carbon (percentage)	0.29
Manganese (percentage)	1.60
Ultimate tensile stress, in pounds per square inch	93 000
Yield point stress, in pounds per square inch	54 000
Proportional limit, in pounds per square inch	40 000
Elongation in 8 in. (percentage)	25
Elongation in 2 in. (percentage)	31
Reduction of area (percentage)	63

Further work on this type of steel, in an endeavor to secure a high elastic limit in a normalized plate, suggested an analysis of 0.30% carbon, 1.30% manganese, and 0.90% silicon.

The effect of combinations of chromium and copper with manganese in a 0.30% carbon steel, has also been investigated at Woolwich Arsenal, the experimental work covering the following ranges of alloy present: Manganese, 0.50% to 1.00%; chromium, up to 1.50%; copper, up to 1.20%; and silicon, less than 0.10 per cent.

The investigators found that chromium added to manganese seems to give no advantage, but that copper from 0.50% to 1.20% improved the mechanical properties to a marked extent, again in normalized plates, the best results being obtained with an analysis of 0.30% carbon, 0.50% Manganese, 0.90% chromium, and 1.20% copper. Steel of this analysis had about the same properties as a straight silico-manganese of 0.30% carbon, 0.90% manganese, and 1.10% silicon, and registered an ultimate tensile stress of 99 000 lb per sq in., a yield point stress of 62 000 lb per sq in., and an elongation of 29% in 8 in.

These studies have evidently resulted in the specification of the so-called Admiralty D steel with composition and properties as follows:

Carbon (percentage)	0.33
Manganese (percentage)	1.10 to 1.40
Ultimate tensile stress, in pounds per square inch	83 000 to 96 000
Elastic limit, in pounds per square inch	38 000
Elongation, in 8 in. (percentage)	17

The development of high-strength steels in Continental Europe with the possible exception of the work of the German Railways Company, has been toward improved ship-plating. There has been a rather limited need for such steel in bridge work, etc. Until lately this was true of English practice also. With the construction of the Sydney Harbor Bridge, in Australia, it was recognized that a decided saving in weight could be effected economically and a high-strength steel was specified. The contractor supplied a silico-manganese type of the following analysis and strength:

Carbon (percentage)	0.30 to 0.40
Silicon (percentage)	0.15 to 0.35
Manganese (percentage)	0.60 to 1.20
Ultimate tensile stress, in pounds per square inch	80 000 to 95 000
Yield point (minimum), in pounds per square inch	45 000
Elongation in 8 in. (minimum)	$\frac{1\,600\,000}{\text{Ultimate tensile stress}}$
Reduction of area (percentage)	35

In this bridge, carbon steel of the 62 000 to 72 000 lb per sq in. grade (carbon unspecified; silicon, 0.10% maximum; and manganese, 0.70% maximum) was used for the approach spans and for all members of the arch in which the calculated thickness of silicon steel would not have provided sufficiently substantial sections. Of 38 000 tons of steel in the arch, 27 000 tons were of the silico-manganese grade. Experience with this steel proved it to be costly to manufacture, and since then a new form of high-strength steel known as Chromador, has been developed. Since its introduction in 1933, this steel has been used quite extensively, more than 25 000 tons being used in 1934. It is a manganese-chromium-copper type of the following analysis: Carbon, 0.30% (maximum); manganese, 0.70% to 1.00%; chromium, 0.70% to 1.10%; copper, 0.25% to 0.50%; and silicon, 0.20% (maximum).

The first attempt at the development of a high-strength steel in Germany occurred about 1912, and the type of steel then introduced was used between 1924 and 1927 by the German State Railways Company and received the specification designation (St 48). It was a plain carbon steel, the carbon range being 0.25 to 0.35 per cent. Experience soon indicated that this steel was unsatisfactory as the higher ultimate and yield strengths were obtained with loss of ductility and shock resistance. Rolling technique was likewise difficult where many different sizes and shapes were required.

Beginning in 1912 there was apparently an attempt to overcome these difficulties by the use of additional silicon, and such steels in German specification nomenclature were designated as the (St Si) steels. One of the more important of this type was the Freund steel known first as "F" and later classed as an "(St Si)" steel, which appeared in 1926. As might be expected (since in Germany steels are generally specified or known by their ultimate tensile strength in kilograms per square millimeter), Freund steel was of quite variable composition with respect to silicon and manganese content.

The typical composition and properties of an earlier Freund steel of this type shows 0.12% carbon, 0.46 to 0.70% manganese, and 0.95% silicon, with a yield point of approximately 50 000 lb per sq in., a tensile strength of 70 000 lb per sq in., an elongation in 8 in. of 25%, and a reduction of area of 60 per cent. All the Freund type steels had a carbon content of less than 0.16 per cent. The American practice for similar types permitted the carbon to increase to 0.30 per cent. The Freund steels examined in America appeared to be excessively dirty and indicated heavy deoxidization with aluminum.

When officially adopted by the German State Railways Company as its (St Si), the Freund steels were required to meet, for thickness up to $\frac{3}{4}$ in., an ultimate strength of 71 000 to 81 500 lb per sq in., an elastic limit of 51 000 lb per sq in., with a maximum carbon content of 0.20 per cent. For thicker sections the maximum carbon content was removed and slightly lower strength values were specified.

This steel type was used extensively, but proved uneconomical despite the low cost of its alloys, on account of high manufacturing cost, large cropping losses (50% to 55%), and a tendency toward red-shortness. Its corrosion resistance was inferior to the plain carbon steel, structural grade (St 37), and, as might be expected when wide variations in analyses are permitted, the yield-point variations of different sections were excessive.

In 1928, the Dortmund Union Steelworks produced the so-called "Union" steel which was intended to overcome these difficulties. It contains manganese, chromium, and copper, together with a little silicon. The copper was added to improve the corrosion resistance of the steel, but from later metallurgical research it appears that the copper contributes to the many remarkable mechanical qualities of this steel. The composition of this new steel is as follows: Carbon, 0.15%; chromium, 0.40%; copper, 0.50 to 0.80%; manganese, 0.80%; and silicon, 0.25 per cent.

TABLE 10.—Specifications for German Steel

Description	Union Steel I	Union Steel II (for export)
Ultimate tensile stress, in pounds per square inch	74 000 to 89 000	80 000 to 94 000
Yield point stress, in pounds per square inch	51 000	53 000
Percentage elongation in 8 in.	20	18

The specifications are presented in Table 10. It is claimed that the steel is no more difficult to melt and roll than mild steel and surprisingly uniform properties are realized over a wide range of sections. Its impact strength is good and it is less susceptible to age embrittlement than (St 37) or (St 48). Short-time tests indicate a high corrosion resistance.

The aforementioned steel, however, is only one of a group of steels known as (St 52); that is, of the 75 000 lb per sq in. grade. Of the other steels only meager information is available. They appear to be of two types: (*a*) (St Si) steel, modified by reducing the silicon and increasing the manganese as in the Lauchhammer steel, sometimes with the addition of copper nominally to improve the resistance at atmospheric corrosion, as the steel of the Mitteldeutsche Stahlwerke; and (*b*) steels introducing copper, molybdenum, and chromium:

(1) Union steel;
(2) Steel of the following content: Chromium, 0.70% to 0.90%; copper, 0.30%; and silicon, 0.30% to 0.50%;

(3) The Krupp steel containing manganese and copper;
(4) The Gutehoffnunghutte Oberhausen steel, which is similar to the steel of the Mitteldeutsche Stahlwerke, with the addition of a small quantity of molybdenum; and
(5) Copper-molybdenum steels.

Apparently, copper was introduced into these German steels to gain corrosion resistance, but it was soon found that it had desirable mechanical effects as well. The Vereinigte Stahlwerke investigated this point specifically with important results. It was found that the simple addition of copper produced a marked increase in the yield point and in the ratio of yield point to tensile strength without the loss of ductility or impact strength found in carbon steels. In these respects silicon and copper are quite similar. The desirable properties of copper in steel are retained and appeared even to be enhanced by the presence of other alloying elements, such as manganese, chromium, and silicon. On the other hand, the best results appear to have been obtained with low carbon.

In 1929, the French adopted a chromium-copper steel very similar to the Union steel. The carbon, silicon, and manganese contents are left to the discretion of the manufacturers who must adjust them to satisfy strength and ductility requirements.

This report necessarily offers only a rapid survey of the development of European high-strength steel. The Sub-Committee believes, however, that it covers the essentials of its development. The history of European steels has been not unlike that of the steels in the United States progressing from the older grades of plain carbon, into the nickel and silicon steels, and then to the silico-manganese types. In their very latest groups, the Europeans have gone, perhaps, somewhat ahead of this country in the use of chromium, molybdenum, and copper for structural steels, but progress along that line is also being made here.

The reasons for this constant change toward a better high-strength steel have been the same abroad as in the United States; namely, excessive cost for resulting gains, or some imperfection in the steels themselves, or both. The present status of the European steels—that is, those which are in considerable use—is presented in Table 11.

Economic Status of High-Strength Steel

This résumé of the present status of high-strength structural steels can be summarized, as follows:

(1) There are quite a number of steel types available, most of which have been used in more or less important structures.

(2) At the present time there are many new types and more complex types being considered.

(3) High-strength structural steels, therefore, may be considered as passing through a transition period and with such a status it is becoming increasingly apparent that not much is known yet about their full properties or their true economy.

TABLE 11.—PHYSICAL AND CHEMICAL PROPERTIES OF HIGH-STRENGTH FOREIGN STRUCTURAL STEELS IN USE AS OF JANUARY, 1936

Country or origin and specification designation	MECHANICAL PROPERTIES			CHEMICAL ANALYSIS						
	Minimum yield point, in pounds per square inch	Ultimate tensile strength, in pounds per square inch	Percentage elongation in 8 in.	Carbon	Manganese	Silicon	Nickel	Chromium	Copper	Remarks
(a) ENGLAND										
Mild steel, BSS No. 15	31 000	63 000 to 74 000	20	18	50	..	..	..	..	As used in Sydney Harbor Bridge
				0.30	0.60	0.15	..	..	...	
Silicon steel	45 000	80 000 to 95 000	16							
				0.40	1.20	0.35	..	..	..	
Nickel steel	50 000	85 000 to 100 000	15	25	0.70	0.12	3.0	..	..	
				0.30	0.70	0.20	..	0.70	0.25	
Chromodor	52 000	83 000 to 96 000	17	Maximum	1.00	Maximum	..	1.10	0.50	
(b) GERMANY										
				0.25	0.50	0.15	..	..	..	Typical chemical analysis for German steels, chemistry not specified
St 48	41 000	68 000 to 82 000	18							
				0.35	0.70	0.30	..	..	..	
				0.12	0.75	0.80	..	..	0.25	
St 52	51 000	74 000 to 88 000	20							
				0.15	0.80	1.10	..	..	0.40	
				0.12	1.20	0.30	..	..	0.30	
St 52 (Krupp)	51 000	79 000 to 91 000	20							
				0.15	1.60	0.60	..	..	0.60	
(c) FRANCE										
Ac 54	51 000	77 000 to 91 000	20	0.15	0.90	0.17	..	0.40 0.60	0.30 0.60	....

Structural engineers have been so conversant with plain carbon steel, have had such a long period of experience with it, and as a rule have designed so conservatively with it, that they now look upon it with assurance. Can these newer grades of higher strength steel be accepted with equal assurance? The Sub-Committee believes not. Again and again, Committee deliberation has brought forward questions and desires for exact knowledge that seem to have no answer because of lack of sufficient knowledge of the mechanical properties of these steels. This condition has been brought about by the necessity of greater economy in the use of steel, which means higher design stresses, thinner sections, more flexibility in structures, and, therefore, greater secondary stresses and greater chances for failure by fatigue, and greater danger of shortened life from corrosion.

It is evident that when an attempt is made to appraise correctly the value or economic status of a high-strength steel, it must be examined searchingly, in all phases of its making and use. So important are many of the questions brought into discussion in Sub-Committee meetings that it does not seem out of place at this time to bring some of them forward for consideration in order that every one may realize the seriousness of the problem, which may be defined as follows: A satisfactory high-strength steel must be economical; to that end it should have a yield stress (perhaps an elastic limit) and an ultimate strength sufficiently high to permit a real saving in weight commensurate with its increase in cost. It should have a high degree of real ductility, high impact, and fatigue resistance. It must be uniform not only from heat to heat but when fabricated into different shapes and thickness of sections. Since higher design stresses mean thinner sections it should be more corrosive resistant than standard carbon steel grades. Its mechanical properties must, if possible, depend on its composition without special forms of heat-treatment, or if the latter is necessary, it should be of some simple form easily controllable, and such as not to interfere with subsequent fabrication. It should not air-harden erratically or flame-cut detrimentally. It must be workable in fabrication and field erection and, to-day, it must be weldable also.

With this general problem in mind an examination of some of its phases in detail should be of interest. For purposes of review they will be discussed under the following headings: (1) Furnace Operations; (2) Rolling Mill; (3) Fabrication; (4) Uniformity and Mechanical Properties; (5) Corrosive Resistance; (6) Their Economic Position in Structures; and, (7) Riveting and Welding.

Furnace Operations.—It is essential that a high-strength steel shall be devoid of critical furnace manipulation so that its analysis and uniformity may be met with assurance. Lost or diverted heats are costly. Its composition should be as cheap as possible from the additive alloy standpoint. It must not greatly shorten furnace or mould life, nor solidify so as to promote segregration or a banded structure of the finished metal. It must not entail excessive cropping and heavy scrap loss, due to solidification with deep piping in the ingot.

Experience with structural silicon steel has shown it to be costly in this respect. Silicon steel is made with more difficulty by the basic open-hearth process, which is, of course, used for production on a commercial scale. In this case the bath contains a much greater amount of active oxide, which oxidizes the silicon. There is a large loss of silicon, therefore, and, in addition, since silica is an active acid at steel-making temperatures, it tends to flux the basic lining. One of the most serious difficulties is the re-phosphorization caused by loss of basicity of the slag. The phosphoric acid cannot be held by any but a strongly basic slag, so it passes into the metal. Another difficulty is the variation in silicon content of metal due to its absorption by basic slag. As a result, when silicon steel is made in the basic open-hearth, a small part of the silicon alloy is added to the furnace and the greater part to the ladle.

Large-scale manufacture of silicon steel in the basic open-hearth is accompanied by certain difficulties which do not harass the producer of carbon steel. First, a purer furnace charge is required, to meet quality demands. The increased silicon and manganese content increase the cost, and lime consumption is higher and fuel consumption is greater than for the manufacture of carbon steel. The high silicon content makes the steel viscous, so that higher melting temperatures are necessary. The higher temperatures decrease refractory life. Greater care must be exercised to avoid contamination with refractory fragments. The high pouring temperatures shorten mould life. Impurities are likely to become entrapped in the viscous metal as it is poured, and there is a tendency to form deep pipes, in some cases requiring scrap losses as high as 50 per cent. To reduce the loss from piping, hot-top ingots are required. There is some silicon segregation. In order to secure the same mechanical properties for sections of different thickness, varying analyses of carbon, silicon, and manganese, are necessary which makes furnace control more difficult.

Many of these same difficulties were found to exist to greater or less extent with the intermediate manganese grade. Hot-top ingots had to be used. To limit segregation in general the ingots were made no larger than necessary for rolling-mill requirements. To meet the mechanical properties specified, the carbon, silicon and manganese content had to be varied, which, to keep the proper relationships, meant close control of furnace and ladle conditions. Ingot top discard of 20%, at first thought sufficient, had to be increased to avoid segregation of carbon in high manganese heats.

These statements are not to be taken as criticisms of any particular type of steel. They are given to show the difficulties with which a steel maker has to contend in working with these newer steels and to point out wherein they become so costly that they are uneconomical. They also show the "why" of the tendency toward a silico-manganese type.

It has been stated that the relative prices of ferro-manganese and ferro-silicon at a given time will determine the mixture adopted by the manufacturer. This is not necessarily true. A high-grade ferro-manganese (80% manganese content) may be cheaper than ferro-silicon (50% silicon content) for the same percentage of alloy addition, but a high-grade ferro-manganese adds to the difficulty of carbon control and if this is avoided by the use of lower grade ferro-manganese then the latter may not prove as economical. The silicon is stated to be usually rather cheaper; but manganese is believed by most American steel-makers to produce less ingot piping and, therefore, cropping loss and greater freedom from rolling tears and slivers, and a better rolling product, even at some higher cost a more economical steel.

Manganese, however, is about as effective a hardening element as carbon, so that if the manganese is increased, care must be taken to keep the carbon down. Carbon and manganese produce increased strength, silicon yields strength with ductility. The balance, then, must lie between the ease and economy of furnace operation and the desired physical characteristics.

If one may judge by the tendency toward other alloying elements it may be questioned whether a balance has yet been struck in the silico-manganese type that is all-sufficient, especially when other considerations are given proper weight. Each of the other alloys have their own requirements from the steel-making end, but mostly on account of the fact that they are added principally in the ladle they are less difficult to control, and their non-economy lies in the excess cost of the alloy itself (as, for example, the 3¼% nickel steels), or in difficulties in rolling and fabrication, or both, or in tendencies toward segregation and banding of the crystal structure.

Rolling.—Thus, the 3¼% nickel steel, with its tight adhering scale, possesses increased roughness of surface, and requires more care and power in rolling. The suggestion of the nickel-manganese type is an effort to gain lower cost with better rolling characteristics. Although manganese does facilitate rolling (the surface in the intermediate manganese steel was better than that secured with nickel and the silicon steels, and red-shortness was not apparent), it can, also, add its own difficulties to the problem, difficulties that may, in cases, offset its advantages.

The higher strength steels are more susceptible to air-hardening, especially if the alloying elements tend to produce carbides. Therefore, they are more subject to detrimental cooling strains. In the intermediate manganese type the latter were of such magnitude that in several cases in heavy gage plates, ruptures occurred in rolling in apparently sound metal. It was necessary to take precautions against abrupt temperature changes. The soaking pits and reheating furnaces were allowed to cool somewhat before charging. Whenever possible the material was worked through from ingot to finished product material without allowing it to become cold. When it was desirable to allow the material to become cold to permit removal of surface defects from slabs before rolling, or because of unavoidable mill contingencies, the slabs were annealed before they became cold. Then, before being charged into the reheating furnaces, they were slightly preheated by placing them near the furnaces for a time.

With high-strength steels subject to temperature strains from irregular cooling, consideration may have to be given to the maximum thickness of section that should be rolled. No doubt, the principal point to be considered will be the volume of individual pieces, then the finished form, and perhaps the use, unless extraordinary methods of handling or treatment are developed.

Knowledge gained with the intermediate manganese steel as furnished for the Bayonne Bridge indicates that better results can be obtained, at least for this steel, if the thickness of large plates is limited to 1 in.

Those elements which confer ductility, depth of hardening, and sluggishness of transformation through the critical range (and all steels should be finished hot-worked above the criticals) confer benefits in this respect as they tend to eliminate the effect of too rapid surface cooling, and increase uniformity of properties throughout the thickness of section. There is also the added danger, in the rolling of the higher-strength steels, of increased roll breakage where mills have been developed for plain carbon steels. The higher strength steels require more power. Furthermore, because they are more susceptible to temperature variations, more care must be exercised in the use of cooling water on the rolls, and in cooling from the rolls. As a rule, the high-strength steels do not come from the rolls in as flat a condition as the lower carbons. This causes increased difficulty in riveting, as a quilting effect causes the earlier driven rivets to loosen.

Fabrication.—High-strength steels have caused increased working and fabrication difficulties which again may be exemplified by the experiences with the silico-manganese group—steels of the type with which American engineers have had most experience. In fabricating operations with the intermediate manganese steel there was very little cold-working other than drilling and planing. Cambered plates in heavy gages could not be straightened with straightening rolls or in a press. Camber was removed from the large thick plates by planing the edges.

Some plates, $1\frac{1}{4}$ in. and more in thickness, cracked at the sheared ends when an attempt was made to flatten a slight shear bow by passing the plates through cold rolls. This bow was caused by the pressure of the movable shear blade on the back ends of the plates where they were not supported by the fixed blade or die. The difficulty was overcome by planning $\frac{1}{4}$ in. from sheared ends of the bowed plates and straightening them in a press instead of by rolling. Subsequently, the bowing was avoided by turning the plates around so that in cutting both ends the main body of the plate rested on the die of the shear.

Structural manganese steel having a yield point up to 70 000 lb per sq in. can be planed in a rotary planer, and can be punched, drilled, and reamed, with the equipment ordinarily used for heavy carbon steel work, but at somewhat lower speeds. High-grade drills must be used, and these must be ground more frequently than for carbon steel. When the yield point exceeds 70 000 lb, there is considerable strain on the machinery in the operations of punching, drilling, and milling, and this work then becomes impracticable.

The surface of flame-cut edges is so hard that planing or milling them in the structural shop is impracticable. Flame-cut specimens are brittle, but when the hardened surface is removed to a depth of $\frac{1}{8}$ in., the tests show normal ductility. Similar precautions have to be taken with silicon steels.

The strength of silicon steel is such that the usual fabricator is not equipped to straighten it as he would expect to straighten "as-rolled" carbon steel. Efforts to cold-straighten heavy plates ($\frac{3}{4}$ in. and more thick) have produced badly cracked edges. One large fabricator prefers to divide plate thicknesses greater than $\frac{7}{8}$ in. into two thinner plates stitch-riveted together, rather than attempt to straighten and "fit up" in one thickness. The mills will probably agree that mill-straightening and attention to camber, sweep, and squareness of section, require more attention than for carbon steel, as otherwise the material will be beyond the fabricator's ability to correct.

As piping is deeper in silicon ingots than in carbon, angles should be ordered long enough for a preliminary shearing (or milling) at each end in search of pipes. All sheared edges should be planed to eliminate cold-worked material. Oxy-acetylene cutting of silicon steel leaves an air-hardened edge which is dangerously brittle. However, a light grind ($\frac{1}{32}$ in.) with a carborundum wheel removes all trace of damage so that the cutting process is permissible when followed with such a grind. This is in contradiction to the flame-cutting of carbon steel of structural grades, which leaves a perfectly ductile edge.

Because of its stiffness, silicon steel is difficult to draw up into perfect contact for riveting "fit up." The best practice is to use no rivets less than 1 in. in diameter. When sub-punched or sub-drilled $\frac{13}{16}$ in., this permits the use of $\frac{3}{4}$-in. fitting-up bolts, which are the smallest that will do effective work. As a corollary to the 1-in. rivets, it is desirable to maintain scrupulously, or, if possible, to exceed, the edge distances (and rivet-center distances) ordinarily specified. No angles with legs less than 4 in. should be used in the design. It is largely in recognition of this situation that the mills have recently changed the 7 by $3\frac{1}{2}$ and 8 by $3\frac{1}{2}$ angles to 7 by 4 and 8 by 4 angles.

Sub-punching or, for that matter, full-sized punching, of material more than $\frac{3}{4}$ in. thick is too great a strain on the usual punching equipment, and a fabricator will do well to avoid it. This fact is leading the fabricators to a re-study of drilling methods for bridge work, as suggested hereafter under the discussions of welding. In this connection it should be recognized that two steels with the same ultimate strengths do not necessarily possess the same ease of fabrication, and with the newer steels current knowledge of this phase is quite incomplete.

Fabrication Costs.—The increased cost of fabricating high-strength steel in a bridge shop must be thought of in two ways: First, cost per piece, and then cost per ton. Cost per piece is increased by: (1) The special effort required to segregate the material and ensure that carbon is not substituted for it; (2) the delays in case of a spoiled piece, because of the impossibility of securing another from shop or mill stock; (3) the increased proportion of sub-punched and drilled work, and of edge planing; (4) the greater breakage of punches, strain on punch frames, and wear on cutting tools; and, (5) occasional hard spots due to segregation which make drilling and milling almost impossible. With all these items of increased cost, it is not believed, however, that they add more than 10% to the fabricated cost of a piece. This figure has been reported by German as well as by American fabricators, as the result of experience with the silico-manganese types.

Of more importance is the difference in cost per ton. If, for example, two alternative bridge chords are assumed to occupy the same position in a truss and, therefore, to require the same rivets, holes, lattice, etc., the high-strength chord will cost about 10% more to fabricate than the carbon chord—but it will weigh about 75% as much. The fabricating cost per ton, therefore, will be 1.10 ÷ 0.75, or 1.47 times that of carbon steel. In making comparative estimates this increase of cost per ton because of loss of tonnage, must not be overlooked. The same general truth holds also for costs of erection, there being the same number of pieces to raise and the same number of rivets to drive, with about 75% of the tonnage to which to charge the cost.

Uniformity and Mechanical Properties.—When one considers the mechanical properties and the uniformity of these properties of the high-strength steels, one becomes most aware of his lack of knowledge. Plain carbon steel has been used as the comparator, the structural engineers feeling assured from extensive experience with it, that if the similar properties of the high-strength steels were known the latter could be evaluated correctly. On this basis it must be admitted that structures have been built successfully with many of the steels discussed in this report. They have short-comings, however, and current knowledge of them is not complete. Furthermore, conditions have developed in the necessity for more economical design, that have forced more critical consideration of their properties.

Are specifications to be only in terms of yield stress, ultimate strength, and elongation as in present German practice, permitting the steel maker to furnish whatever analysis he wishes that will give the properties specified? Is it necessary, as many engineers believe, to know the elastic properties? Should a steel have a high ratio of yield stress to ultimate strength, or should it have a high ratio of elastic limit to yield stress and a low ratio of yield to the ultimate? Shall chemical analysis be specified also—and if so shall relatively wide limits be specified for each element so that in reality a series of grades of steel will be secured rather than a particular type, or should an attempt be made to find a steel analysis that in thick and thin sections will have identical properties without recourse to changing analysis? If wide changes in analysis are to be permitted, will all the steel furnished under such grade have identical properties? Is it possible to secure uniform properties, other than analysis, by ordinary rolling procedure, or must resort be had to temperature control, especially control of the rate of cooling from the rolls? Will normalizing or stress relief at low temperatures become inevitable for uniformity or desirable because of additive value in itself? These are some of the questions that must be answered before one can determine how uniform in properties the steels are to be, and, therefore, they are part of the criteria for evaluating the various alloying elements.

To-day, the engineer specifies elongation per gage length, reduction of area, and bend testing, although the latter two are either being neglected or made less severe. Are those properties sufficient indication of the toughness of high-strength steels or does the engineer need to know more about what one investigator terms the "crackless stress propagation" quality or, as others specify it, the "inherent ductility"? Here, again, the experience of engineers with plain carbon steel has taught that it does possess to large degree the ability to relieve local over-strain. The tangential stress at the edge of a hole in a plate—a rivet hole or a pin hole in a built-up eye-bar—may be as much as three times the average uniform stress in such a plate under tension, and if the metal possesses the ability to flow and relieve and dissipate this over-stress the member will be safe. If, however (as has been proved in some of the high-strength steels) the material apparently lacks this quality the metal at such position of over-stress may form a crack which will progress through the piece although the metal appears to have, as determined by the ordinary tension test, the requisite ductility. If this property must be determined by what means shall it be measured?

Designers compute and make allowances for impact on structures; yet they do not specify directly an impact resistance test, assuming elongation, reduction of area, and the bend test requirements to be sufficient. Recent studies indicate that impact resistances of steels are not identical at sub-zero temperatures with those at normal temperature, and it may be necessary to consider more seriously those alloys that produce fineness of grain in the metal and enhance impact resistance.

Tensile tests are usually taken with the direction of rolling; yet for gusset-plates it is important that the material have equal ductility transversely—that is, across the direction of rolling. At times, steels show rather marked differences in this respect.

With higher strength steels and design stresses, there is no gain in stiffness of structure because for all the steels the measure of its stiffness, the modulus of elasticity, is practically identical. Structures are then more flexible. Increased deflection in the lighter weight members increases secondary stress, and their lower inertia makes them more subject to impressed vibrational stress. Fatigue resistance, therefore, becomes of more importance.

In practically all fatigue work to date, the effort has been devoted to determination of the so-called "fatigue limit", the stress that will withstand at least 10 000 000 reversals of stress from a positive to a negative value (of equal degree; that is, from a definite stress in tension to an equal stress in compression). It is assumed as a practical expedient that if the material will withstand 10 000 000 reversals it will withstand an infinite number; and in connection with this work considerable research has been done on the effect of range of stress, either all in tension or all in compression.

In many structures, however, a requirement to withstand 10 000 000 reversals may be unnecessarily severe. It would appear that the civil engineer is more interested in the envelope of fatigue properties comprising the effect of range and reversals, probably of not more than 2 000 000 cycles in number. What he will wish to know is how many stress cycles the material may be expected to withstand at a definite stress range. Such information does not seem to be available for high-strength structural steels.

Corrosion Resistance.—Higher design stresses, permitting thinner structural sections, give cause for more thorough consideration of the corrosion resistance of the higher strength steels. If one were to assume, for example, in the design of the web of a chord member, that the permissible working stress for plain carbon and for silicon structural steel was in the ratio of 15 000 to 20 000 lb per sq in. (that is, in the ratio of 3:4), the web-plates, if made the same depth for both steels, would have a respective thickness of the inverse ratio; that is, 4:3. A $\frac{1}{2}$-in. and a $\frac{3}{8}$-in. plate might be representative of the condition. Assuming $\frac{1}{32}$ in. of corrosion on each side of each plate, the $\frac{1}{2}$-in. carbon-steel plate would still possess $87\frac{1}{2}\%$ of its original section while the $\frac{3}{8}$-in. silicon plate would have $83\frac{1}{4}\%$ of its original section. The thinner sections of high-strength steels for equal life would require increased maintenance in painting and better protection against corrosion.

It is quite generally agreed that the protection against atmospheric agencies afforded by the various metals is due to the formation of thin tightly adherent oxide films which act as buffers against further corrosive attack. The so-called copper-bearing steels are examples of this type of protection, as is the anodic treatment of aluminium. Such protective films are of real value only when, having once formed, they can remain undisturbed. If they are continually abraided, necessitating constant replacement by further corrosive action, they are not nearly as valuable as a protecting agent. In most locations in structures such films will be protective.

The alloys available for this protective purpose from the structural steel standpoint are chromium, copper, and nickel, all of which not only have excellent corrosion resistance in themselves under many corrosive conditions, but have the further advantage, as a constituent of steel, of forming a fine-grained, tightly adherent, corrosion product. It is recognition of this quality plus experience which indicates that the silico-manganese types of steel are not as corrosive resistant as the older low-carbon steels, that has caused European steel makers to incorporate one or more of these elements in their analyses, in addition to their recognized value as alloys for steel.

The Economic Position of High-Strength Steels.—High-strength steels should not be used indiscriminately. Consideration should be given to their fabrication costs (which include the extra cost of manufacture) and their real economic position as part of a structure and for the structure as a whole.

The Individual Member.—Relative costs of a single member, as a beam or girder, or truss element, require individual determination. This may be exemplified by the following computation of a typical case, that of a compression chord of a bridge, 42 ft long with a slenderness ratio equal to 30, assuming silicon-steel at a mill extra of $12 per ton over carbon steel:

Carbon Steel		Silicon Steel	
$p = 15\,000 - \frac{1}{4}(30)^2$	= 14 775	$p = 20\,000 - 0.46\,(30)^2$	= 19 585
590 000 lb @ 14 775.. required gross section	= 40.0 sq in.	590 000 lb @ 19 585.. required gross section	= 30.2 sq in.
42 × 3.4 × 40...... of main material	= 5 710 lb	42 × 3.4 × 30.2.... (silicon steel)	= 4 310 lb
Details, say, 40%...	= 2 290	Details .... (carbon steel)	= 2 290
Total	= 8 000 lb	Total	= 6 600 lb

Carbon Steel	
Plates and shapes, 7 400 @ $0.018	= $133.20
Rivets, 600 lb @ $0.028	= 16.80
Material at works	= $150.00
Drawings	= 20.00
Manufacturing	= 85.00
Freight to destination @ $0.25	= 20.00
Erection, say, $24 per ton	= 96.00
Total cost of chord	= $371.00

Silicon Steel	
Plates and shapes:	
4 310 lb @ $0.024	= $103.40
1 690 lb @ $0.018	= 30.40
Rivets, 600 lb @ $0.028	= 16.80
Material at works	= $150.20
Drawings	= 20.00
Manufacturing same as carbon + 10%	= 93.50
Freight to destination @ $0.25	= 16.50
Erection, say, $24 per ton	= 96.00
Total cost of chord	= $376.20

This tabulation shows that the difference in the total cost of a member is negligible and doubtful, depending upon the conditions, known or guessed at, of the specific case. A similar comparison for a single, simple beam, with no details to be estimated might be as follows:

Carbon Steel	
1 448 000 in-lb @ 18 800 lb per sq in.	= 80.4
(Use 15-in. I-beam @ 60.8 lb	= 81.2)
Length, 25 ft @ 60.8 lb per ft	= 1 520 lb
Material @ $1.80	= $27.40
Drawings	= 2.00
Manufacturing	= 6.00
Freight @ $0.25	= 3.80
Erection, say	= 9.00
Total	= $48.20

Silicon Steel	
1 448 000 in-lb @ 24 000	= 60.3
(Use 15-in. I-beam @ 45 lb	= 60.5)
Length, 25 ft @ 45 lb per ft	= 1 125 lb
Materials @ $2.40	= $27.00
Drawings	= 2.00
Manufacturing, same as carbon + 10%	= 6.60
Freight, @ $0.25	= 2.80
Erection, say	= 9.00
Total	= $47.40

In this instance the scale is turned by the item of freight. As long as the silicon mill extra remains approximately $12 per ton there is extremely little, if any, saving by its use, individual members considered.

The Assembled Structure.—All buildings, and all short-span bridges, are little affected in their design calculations by the weight of the steel alone, and the foregoing considerations hold. It has already been noted that in a few buildings, the silicon steel premium has been paid in order to reduce the space occupied by the framing.

As bridge spans increase, and the stresses due to dead weight of steel become an increasingly large proportion of the total (dead load plus live load plus impact) stresses, the saving of a certain percentage (25% or 30%) in the weight of each individual member produces a sensible diminution of the maximum stress in most members, therefore, further reducing their required areas. (Posts and hangers loaded by a single panel are excepted.)

Therefore, a span is eventually reached at which the saving in first cost by the use of high-strength steel in the trusses (sometimes girders) outweighs any contrary considerations. As it has just been exemplified, using silicon steel as the basis of comparison, the first cost is just about a "stand-off" for very short spans. It follows that the span lengths at which to change from carbon to silicon steel is a function not so much of first cost, as of the individual engineer's attitude toward the "contrary consideration." The divergencies of attitude that exist or have existed, may be sufficiently illustrated by two instances: (1) The 400-ft and 600-ft double-track spans of the Castleton Bridge (1923) were built of carbon steel; and (2) the truss approach spans to the Ambassador Bridge (1927) as short as 152 ft were built with silicon steel chords.

It may prove helpful in this respect to give a series of recommendations intended to apply to silico-manganese steels, but which no doubt are equally applicable to any high-strength steel. They are intended to apply to buildings, bridges, etc., but not to structures subject to higher ratios of shock, vibrations, and stress reversal than railway bridges. They are of course quite general and may be ignored under exceptional circumstances. It is recommended:

1.—That silico-manganese steel be not utilized for building frames; crane runways; beam bridges; beam floor systems; plate-girder spans of less than 90 ft; railway floor systems; bracing systems; and detail material, such as tie-plates, lattice, stiffeners, etc. Reasons: The small savings (if any) theoretically possible on some of these items do not compensate for difficulty of procurement on short notice, increase of deflection due to higher unit stress, and decreased margin for corrosion on account of thinner material. Of two adequate structures of nearly equal cost, the heavier one is the wiser purchase.

2.—That silico-manganese steel be used for long girders in which deflection is not a factor and flanges are difficult to build up in carbon steel without excessive rivet grips; for trusses (main material and gussets) of highway spans longer than 200 ft and of railway spans longer than 300 ft (usually excepting the one-panel posts and hangers); for bracing systems which participate in chord stress due to deformations; and for most parts of movable bridges, derrick booms, crane bridges, and other structures in which the power required to overcome inertia is an important factor.

3.—That designs in silico-manganese be kept simple: The smallest possible number of rolled sections should be called for; substantial quantities ordered at any one time (30 tons or more from any one set of rolls); and a consistency in design be carried out incorporating the silico-manganese steel throughout all the chord members or throughout all the web diagonals if it is necessary to incorporate it in any one.

In other words, it is felt to be for the good of the profession and of the industry, that high-strength steels be regarded as special and be not ordered indiscriminately, in small and scattering quantities, or for no good reason.

Riveting and Welding.—No survey of high-strength steels may be considered complete without reference to methods of joining individual pieces to make up the units of the whole. Riveting and its new competitor, welding, should be considered.

When silicon steel meant steel high in silicon, it was admitted that this was not a forging steel, and not satisfactory for rivets. Universally, therefore, the regular carbon-steel rivet has been used for the fabrication of silicon-steel bridges. The result is a loss of the potential economy of the stronger steel, as the size of gussets and the number of splicing and connection rivets are both unchanged from what they would be in a carbon-steel design.

Nickel-steel rivets have been used with nickel structural steel; but they have given trouble in driving and have not proved entirely satisfactory. This latter statement applies also to certain types of silico-manganese rivets that have been tried.

Part of the trouble has been lack of forging quality of the rivet material, but there are other elements in the problem that must be considered. To begin with there is lack of planeness in high-strength plates which makes fitting up difficult and causes a quilting action as riveting proceeds, which is productive of loose rivets. The latter is especially noticeable in field splices where it is difficult to pull pieces together. Then there is the question of the heating of rivets and the thermal expansion characteristic of individual rivet materials when of alloy type. Alloy rivets must be thoroughly soaked through to a uniform temperature in heating, or they will not upset properly. Consideration must be given to the temperature of driving, the duration of driving, and to air-hardening characteristics. The mass of cold metal surrounding a heated rivet has a decided quenching effect on the rivet metal structure and may make forging difficult; or it may leave the rivet highly stressed from sudden cooling, or hard and lacking in ductility. Non-recognition of these facts quite probably has been responsible in some measure for lack of success to date with the alloy steel rivets, and hence the loss of potential economy through the necessitous use of the carbon rivet, which is free from most of these faults.

The ship-builders have been more alert in this particular. As the steel has passed from high-silicon to high-manganese, they have found that the latter does make a good high-strength riveting material, and they have used it in large quantities and have done considerable research work to establish its best composition. The same has been done by German steel makers. Recently, an apparently satisfactory rivet has been developed for use with Cromansil steels. This short-coming in the structural engineering field has been recognized. At present, the American Society for Testing Materials is undertaking to remedy this situation and undoubtedly in the near future, as a result of Committee work now in progress, that Society will adopt a rivet specification corollary to its A94 specification.

With the increased adoption of welding in structural engineering, high-strength steels must now be considered for weldability. Silicon steel is not listed as a "Steel Suitable for Fusion Welding" in A. S. T. M. Specification A151–33. The inference may be drawn from that specification, that if carbon is held to 0.35% or less, and the proper technique is followed, silicon steel may be weldable. That is also reported as the experience of some fabricators.

In the fabrication of the towers for the Golden Gate Bridge it was desired to tack-weld shaft angles and web-plates together so that the full thickness could be drilled from the solid, simultaneously, with no subsequent separation before riveting. The question was raised as to whether this tack-welding would harmfully embrittle the silicon steel, which was of the type: Carbon, 0.27%; manganese, 1.20%; and silicon, 0.24 per cent. Specimens were tack-welded together, therefore, broken apart, and bent; other specimens were drilled and bent until cracks appeared around the drilled holes. The broken weld edges would bend farther than the drilled holes before cracks appeared. Other plates were tack-welded on the edges, drilled, riveted, and bent. The rivets sheared before any cracks appeared in the welds. It was proved, therefore, that tack-welding was not detrimental to the structure in comparison with the other acceptable processes of fabrication. This experiment was not intended to, and did not, touch on the question of welding for transmission of stress.

The U. S. Navy Department has definitely gone on record against the use of the silicon-manganese types of steels for welded, stress-carrying members. Air-hardening steels appear to be decidedly dangerous for this type of fabrication; and it will be recalled that this same characteristic has been shown to be detrimental in flame-cutting structural sections in fabrication. Perhaps, also, it should be considered in field erection where the torch is used to cut out improperly driven and loose rivets. The high temperatures developed at the perimeter of a rivet hole by flame-cutting and the sudden quenching by the mass of metal surrounding the hole may prove as detrimental in an air-hardening steel, if the torch is improperly or carelessly used, as severe cold-working from cold punching.

There has been some successful welding of high-strength steels, of the nickel and Cromansil types when proper technique and heavy covered electrodes of proper analysis have been used, but, on the whole, the characteristics of a structural high-strength steel for important stress-carrying members for welded construction have not been determined. The U. S. Navy Department, in co-operation with a group interested in high-strength steels and welding, is now at work at this problem, and important recommendations may be expected shortly.

SUMMARY

In presenting this report the Sub-Committee is cognizant of the fact that there are in existence to-day a number of steel types that have been used successfully in important structures. It also recognizes that newer types are constantly being suggested. However, it does not feel that, at this time, it should make any definite recommendation toward the formulation of a specification for a particular steel type or make suggestions affecting its use, especially in the light of the considerable lack of exact knowledge concerning certain characteristics of some of these steels.

The very nature of this report has made it advisable to identify several of the alloys under the popular names by which they are known to structural engineers and manufacturers. In doing so, the Sub-Committee selected typical examples and, possibly, did not include all trade names available. Throughout the report a serious effort has been made to avoid advocating special interests, with the understanding that discussers, likewise, will confine their comments to the intended scope of the report as bearing on general classes of alloys rather than on individual proprietary types within those groups.

ACKNOWLEDGMENTS

In this Sub-Committee report, the section on "Manganese Steel" has been largely based on a preliminary report by Messrs. Rys and Troelsch; the section on "Silicon Steel", on a preliminary report furnished by Mr. Jones; and the section on "Foreign Steels", on a preliminary report by Mr. Frankland.

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January 16, 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

WIND-BRACING IN STEEL BUILDINGS[1]

FIFTH PROGRESS REPORT OF SUB-COMMITTEE NO. 31
COMMITTEE ON STEEL
OF THE STRUCTURAL DIVISION

Introduction

Following the publication of its Third Progress Report,[2] the Sub-Committee received a number of helpful discussions, in spite of the fact that the lack of funds prevented their being printed in *Proceedings.* The Fourth Progress Report having been necessarily very brief,[3] any reference to the comments of those who discussed the Third Report have been reserved for the present report. In it the matters considered are: (A) Comments on the discussion of the Third Progress Report; (B) wind force on rounded and sloping roofs; (C) allocation of wind force to windward and leeward walls; (D) simplified methods of determining wind moments; and (E) experimental studies of the behavior of wind bents.

(A) Comments on the Discussion of the Third Progress Report

(1).—Prescribed Wind Force.—Although admitting that designs made in accordance with the Sub-Committee's recommendations taken in their entirety would have the necessary strength without waste, except in the matter of anchorage, Albert Smith, M. Am. Soc. C. E., thinks that it is wrong in principle to use a moderate prescribed wind force and conservative permissible stresses. He would specify a larger wind force and higher working stresses. His is merely an alternative method to that suggested in the Third Progress Report. It is doubtful whether this manner of dealing with the matter would deter the adventurous any more than the type of regulations proposed by the Sub-Committee.

(2).—Permissible Stress for Combination of Wind with Other Loads.—In its Third Progress Report the Sub-Committee accepted as a good one the method suggested by L. E. Grinter, Assoc. M. Am. Soc. C. E., by which allowance is made for the combination of wind stress with other kinds of stress

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NOTE.—Discussion on this report will be closed in **August, 1936,** *Proceedings.*

[1] Presented at the meeting of the Structural Division, New York, N. Y., January 16, 1936.

[2] *Proceedings*, Am. Soc. C. E., December, 1933, p. 1601.

[3] *Civil Engineering,* March, 1934, p. 153.

by neglecting the wind stress up to $33\frac{1}{3}\%$ of the other stresses and proportioning for the excess wind plus dead load and live load at the basic working stress for dead load and live load only. Professor Grinter objects to allowing an increase in permissible stress in rivets and bolts of $33\frac{1}{3}\%$ above basic stresses when no increase is allowed in main wind members, on the ground that at basic working stresses the test factor of safety of a rivet is only about 14% greater than that of the tension member. He overlooks the fact that there may be a compression diagonal in the same panel with a test factor of safety at a basic working stress only 62.5% of that for the rivets. In other words, the compression member is relatively weaker than the rivets, in the ratio of $1.33 \times 0.625 = 0.83$.

(3).—Stability.—Mr. Smith believes that a stability factor of safety of 1.5 against the wind load as specified by the Sub-Committee is inadequate. Others (as, for example, H. V. Spurr, M. Am. Soc. C. E.,[4]) hold that it is sufficient. The matter will be carefully reviewed for the Final Report.

(4).—Allocation of Wind to Braced Bents.—Both in correspondence with the Sub-Committee and elsewhere,[5] Mr. Smith has suggested a very promising method of allocating wind to braced bents for unsymmetrical buildings, or for buildings unsymmetrical for a portion of their heights. Although the Sub-Committee has given some consideration to the problem, it is not prepared to make definite recommendations at the present time.

(5).—Spurr Method of Design.—Dr. Robins Fleming criticizes the method of equal deflections advanced[6] by Mr. Spurr, as lacking in simplicity and recommends either the cantilever or the portal method for moderate ratios of height to breadth. Although sufficient aggregate strength may be secured in this way, there is no assurance that a balanced design will be attained, or that unduly high local stresses may not exist. Mr. Smith and Professor Grinter call attention to the fact that the Spurr method is not applicable to frames in which columns are omitted in certain stories, in which girders are omitted in certain bays, or to any bent in the region of a set-back.

The Sub-Committee is aware that no formal method can be applicable, without modification, to all the unusual or irregular cases that may arise in practice. Unpredictable irregularities that cannot be covered directly by any regulations or code will occur, and for these special treatment will be necessary. No matter what procedure the designer normally follows, he must revert, in such a situation, to the fundamental principles of mechanics and elasticity.

(*B*) Wind Forces on Rounded and Sloping Roofs

It has been admitted for some time that the wind forces generally prescribed in specifications and building codes for the design of roofs of the rounded, and even of the gable, type have been altogether misleading. Except in the cases of certain important rounded roofs of the hangar type, for which the adopted wind forces used have been in conformity with the results of research, the prescribed force has commonly been merely a pressure on the windward slope with no force at all on the leeward slope. This is highly erroneous. The results of many years of experimentation have shown that even in the case of the common gable roof the force on the windward slope may be either partly pressure and partly suction, or entirely suction, while on the leeward slope a large suction may exist.

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[4] *Proceedings*, Am. Soc. C. E., August, 1932, p. 1126.

[5] *Journal*, Western Soc. of Engrs., February, 1933, p. 1.

[6] "Wind Bracing," by Henry V. Spurr, N. Y., McGraw-Hill Book Co., 1930.

In order to establish a logical basis for design an examination has been made of all available research results pertaining to rounded and gable roofs and these results have been plotted for purposes of study in Figs. 1 and 2. To facilitate the presentation of the results for rounded roofs the observed forces have been plotted with respect to the angular distance from the windward eave, a representative circular arc being swept through the two eaves and the ridge. Where buildings of this class have sloping side-walls, or where the roof starts at ground level, the arc has been drawn to connect the two bases and the ridge, as shown in Fig. 1(*h*), Fig. 1(*i*), and Fig. 1(*j*). For consistency, the angular method of representation has also been used for the gable roof in Fig. 2.

The experimental data utilized in the study are those made available by Allard[7], Arnstein[8], Bounkin and Tcheremoukhin[9], Coupard[10], Dryden and Hill[11], Eiffel[12], Flachbart[13], Irminger and Nokkentved[14], Junkers[15], Schoemaker and Wouters[16], Smith[17], Sylvester[18], and Vandeperre[19]. Applicable results from other sources doubtless exist, but so far they have not been available.

In general, the results plotted have been those found for the median, or middle, section of the building; that is, the one equi-distant from the two ends. It is believed that the wind distribution at this section will represent the total force on the building better than any other. The wind has been assumed as acting at 90° to the side-walls, although in some instances slightly greater forces are found to be exerted on the roof where the wind is inclined at a lesser angle to the walls. The most important case of the kind is that for Eiffel's Belfort model, noted on Fig 1, for which a wind at an angle of 60° to the side-walls produces considerably greater normal forces than the 90° wind.

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[7] Premier Congrès International du Béton et du Béton Armé, Liége, 1930, Vol. I, pp. 390–392.

[8] *Transactions*, Am. Soc. Mech. Engrs. (Aeronautics), Vol. 50, No. 4, 1928; also, *Engineering News-Record*, July 24, 1930.

[9] *Transactions*, Central Aero-Hydrodynamical Inst., U. S. S. R., No. 35, 1928. (In Russian.)

[10] *Bulletin* de la Chambre Syndicate des Industries Aéronautiques de France, 1927; also, *Science et Industrie*, Vol. 18 (Construction et Travaux Publics), No. 16, April, 1934.

[11] *Research Papers Nos. 301 and 545* U. S. Bureau of Standards.

[12] “Nouvelles Recherches sur la Résistance de l'Air et l'Aviation”, Paris, 1914.

[13] Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen,” IV Lieferung, 1932, pp. 128–134.

[14] “Wind Pressure on Buildings,” Copenhagen, 1930, Publications, International Assoc. for Bridge and Structural Engineering, Vol. I, 1932; also *Journées Scientifiques et Techniques de Mécanique des Fluides* de l'Institut de Mécanique des Fluides de l'Université de Lille, France.

[15] Correspondence with the Laboratorium der Junkerswerke, Dessau, Germany.

[16] *Het Bouwbedrijf*, October 21, 1932.

[17] *Journal*, Western Soc. of Engrs., December, 1912, and April, 1914.

[18] *Bulletin No. 31*, Engineering and Science Series, Rensselaer Polytechnic Inst., Troy, N. Y.

[19] *Annales des Travaux Publics de Belgique*, Vol. 35, Nos. 2 and 4, 1934.

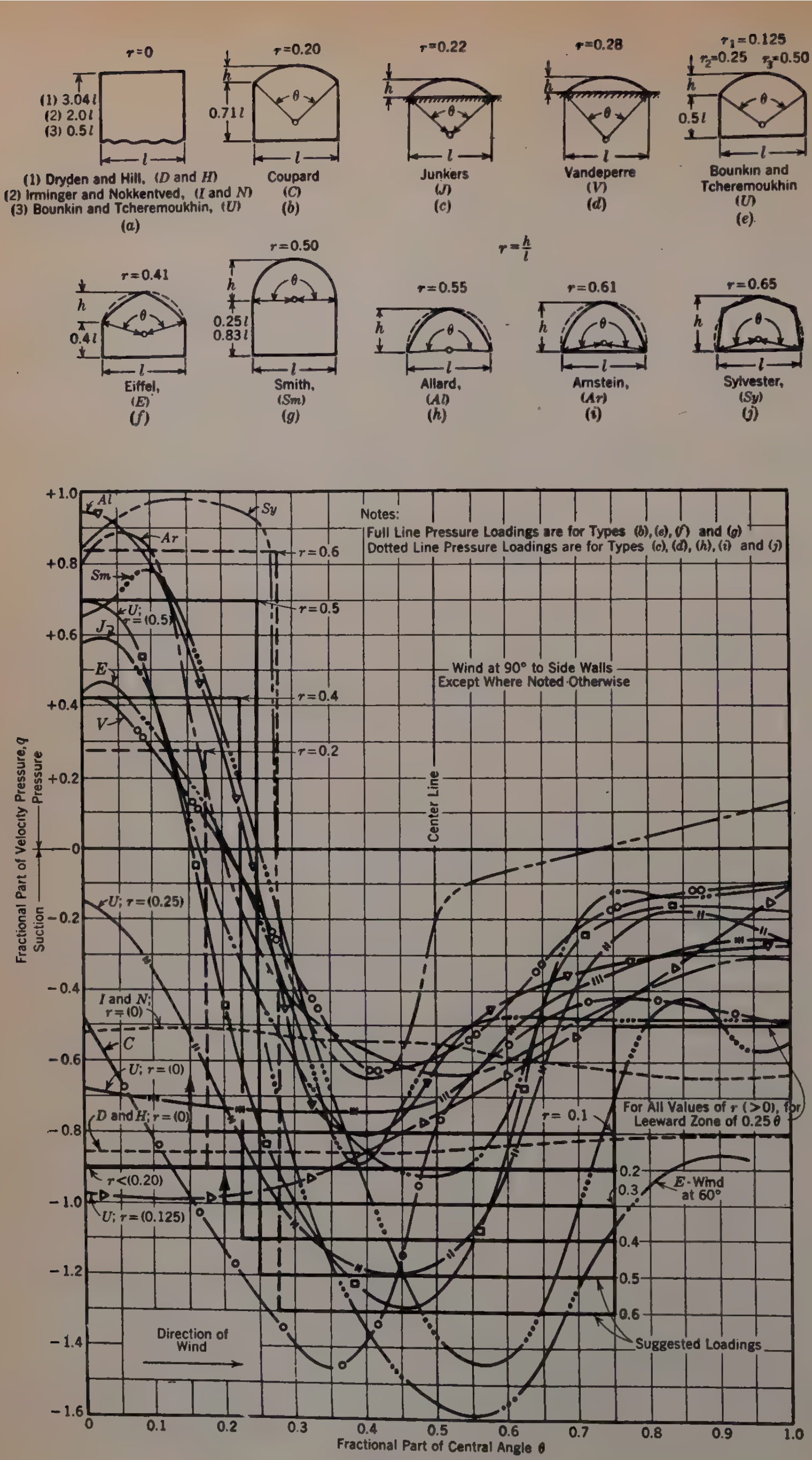
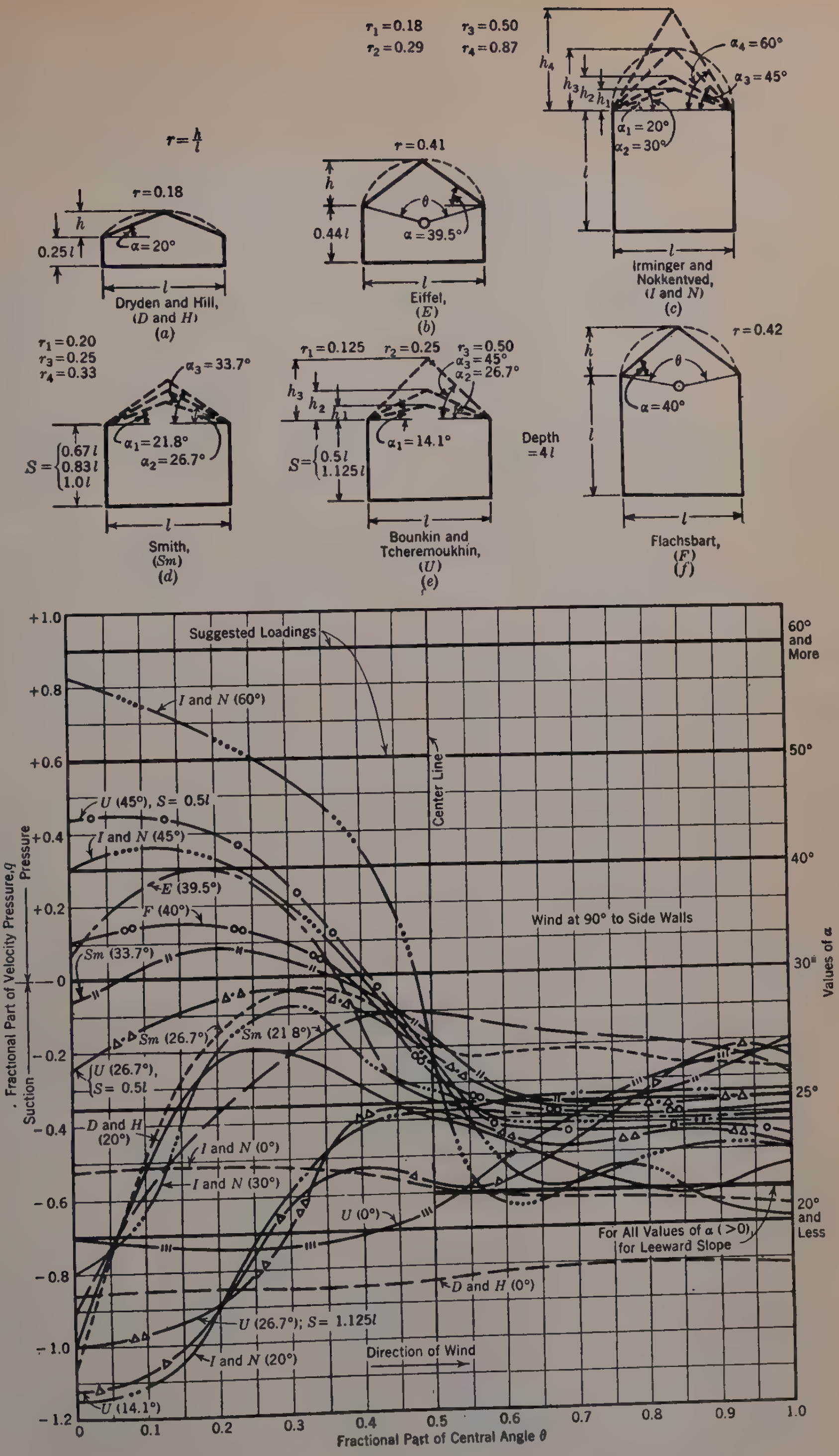

τ=0
(1) 3.04l
(2) 2.0l
(3) 0.5l
l
(1) Dryden and Hill, (D and H)
(2) Irminger and Nokkentved, (I and N)
(3) Bounkin and Tcheremoukhin, (U)
(a)
τ=0.20
h
0.71l
θ
Coupard
(C)
(b)
τ=0.22
Junkers
(J)
(c)
τ=0.28
Vandeperre
(V)
(d)
τ1=0.125
τ2=0.25 τ3=0.50
0.5l
Bounkin and Tcheremoukhin
(U)
(e)
τ=h/l
τ=0.41
0.4l
Eiffel,
(E)
(f)
τ=0.50
0.25l
0.83l
Smith,
(Sm)
(g)
τ=0.55
Allard,
(Al)
(h)
τ=0.61
Arnstein,
(Ar)
(i)
τ=0.65
Sylvester,
(Sy)
(j)
Notes:
Full Line Pressure Loadings are for Types (b), (e), (f) and (g)
Dotted Line Pressure Loadings are for Types (c), (d), (h), (i) and (j)
Al
Sy
Ar
Sm
U; τ=(0.5)
J
E
V
τ=0.6
τ=0.5
τ=0.4
τ=0.2
Wind at 90° to Side Walls Except Where Noted Otherwise
Center Line
U; τ=(0.25)
I and N; τ=(0)
C
U; τ=(0)
D and H; τ=(0)
τ<(0.20)
U; τ=(0.125)
τ= 0.1
For All Values of τ (>0), for Leeward Zone of 0.25 θ
0.2
0.3
0.4
0.5
0.6
E-Wind at 60°
Suggested Loadings
Direction of Wind
Fractional Part of Velocity Pressure, q
Suction
Pressure
+1.0
+0.8
+0.6
+0.4
+0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fractional Part of Central Angle θ

FIG. 1.



$r=\frac{h}{l}$
$r=0.18$
h
$0.25l$
$\alpha=20°$
l
Dryden and Hill, (D and H)
(a)
$r=0.41$
h
$0.44l$
θ
$\alpha=39.5°$
l
Eiffel, (E)
(b)
$r_1=0.18$ $r_3=0.50$
$r_2=0.29$ $r_4=0.87$
h_4 h_3 h_2 h_1
$\alpha_4=60°$
$\alpha_3=45°$
$\alpha_1=20°$
$\alpha_2=30°$
l
Irminger and Nokkentved, (I and N)
(c)
$r_1=0.20$
$r_3=0.25$
$r_4=0.33$
$\alpha_3=33.7°$
$S=\{0.67l, 0.83l, 1.0l$
$\alpha_1=21.8°$
$\alpha_2=26.7°$
l
Smith, (Sm)
(d)
$r_1=0.125$ $r_2=0.25$ $r_3=0.50$
$\alpha_3=45°$
$\alpha_2=26.7°$
h_3 h_2 h_1
$S=\{0.5l, 1.125l$
$\alpha_1=14.1°$
l
Bounkin and Tcheremoukhin, (U)
(e)
Depth $=4l$
$r=0.42$
h
θ
$\alpha=40°$
l
Flachsbart, (F)
(f)
+1.0
+0.8
+0.6
+0.4
+0.2
0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
Fractional Part of Velocity Pressure, q
Pressure
Suction
Suggested Loadings
I and N (60°)
U (45°), S = 0.5l
I and N (45°)
E (39.5°)
F (40°)
Sm (33.7°)
Center Line
Wind at 90° to Side Walls
Sm (26.7°)
Sm (21.8°)
U (26.7°), S = 0.5l
D and H (20°)
I and N (0°)
I and N (30°)
U (0°)
For All Values of α (>0) for Leeward Slope
D and H (0°)
U (26.7°); S = 1.125l
Direction of Wind
I and N (20°)
U (14.1°)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fractional Part of Central Angle θ
60° and More
50°
40°
30°
25°
20° and Less
Values of α

FIG. 2.

Fig. 1 shows the external pressures found by various observers for roofs of round, or generally rounded, form. The intensities plotted are the differences between the actual force intensities existing on the outside surfaces of the model and atmospheric pressure. Where the internal pressure is approximately atmospheric, as for a building with very little air leakage, the plotted intensities may be taken as a basis for the design wind load. If the internal pressure differs substantially from atmospheric pressure, the design wind load must be adjusted correspondingly.

For convenience, the results are shown in terms of the velocity pressure,

$$q = \frac{1}{2} m V^2 \dots\dots\dots\dots (1)$$

in which m = the mass of a unit volume of air; and V = the velocity of the wind in units corresponding to m. For average conditions, and where the wind velocity is expressed in miles per hour, the velocity pressure, in pounds per square foot, may be taken as,

$$q = 0.002558 \; V^2 \dots\dots\dots\dots (2)$$

Examination of Fig. 1 shows that for the usual rise ratios the wind exerts an external pressure on approximately the windward quarter of the arc of the roof and an external suction on the remainder. For a wind at an angle of 90° to the side-walls, the semi-circular roof (Smith's model) develops the greatest suction, considerably greater, indeed, than for Arnstein's streamlined Akron model which has a rise ratio of 0.61.

In proposing a tentative prescription of wind forces on rounded roofs in terms of the rise ratio the Sub-Committee has considered the recommendations of Mr. Albert Smith for semi-circular roofs[20] and the regulations of the Unified Standards for Building Design (Wind Load) issued in 1935 by the Standards Committee of the Union of Socialistic Soviet Republics. The latter are based in part on tests conducted by K. Bounkin and A. Tcheremoukhin in 1924–25 in the laboratory of the Central Aero-Hydrodynamical Institute.[9]

Although a greater body of experimental data will be required before a final prescription of wind load for rounded roofs can be made, it is believed that a general improvement in design and increased safety would be attained by the tentative adoption of the following loadings:

I.—Pressure Loading.—On a fraction, k, of the arc of the roof given, where $r = \frac{h}{l}$, the rise ratio of the arc, by the formula,

$$k = 0.25 \, r + 0.125 \dots\dots\dots\dots (3)$$

and lying adjacent to the windward edge, a pressure as follows:

(1) Where the roof rests on elevated supports, and where r is equal to or greater than 0.25,

$$p = (2.8 \, r - 0.7) \, q \dots\dots\dots\dots (4)$$

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[20] *Journal*, Western Soc. of Engrs., April, 1914, p. 377.

(2) Where the arc of the roof starts from the ground level, a pressure of 1.4 r q.

II.—Suction Loading.—

(1) For the windward zone defined by Equation (3) and applicable to rounded roofs resting on elevated supports with rise ratios, r, less than 0.20, a suction of 0.9 q.

(2) For the values of r between 0.20 and 0.35, the other conditions being as in Paragraph (1), a suction of,

$$p = (2.1 - 6r)\,q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

(3) From the inner edge of the windward zone to a point one-quarter of the length of the arc from its leeward edge, for either type of support, a suction of,

$$p = (r + 0.7)\,q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

When $r = 0$, that is for a flat roof, a suction of 0.7 q is to be considered as applied to the entire roof surface.

(4) On the leeward quarter of the arc of the roof, for all values of r, greater than zero, and for either type of support, a suction of 0.5 q.

The proposed pressure units for the windward zone are somewhat more severe than those specified by the U. S. S. R. Standards Committee, particularly when the roof arc starts from ground level, as will be seen from Fig. 3. This, however, appears desirable in view of the tests results plotted in Fig. 1. The suction proposed for the central portion of the roof is very nearly the same as in the Russian regulations, but the suction for the wind-

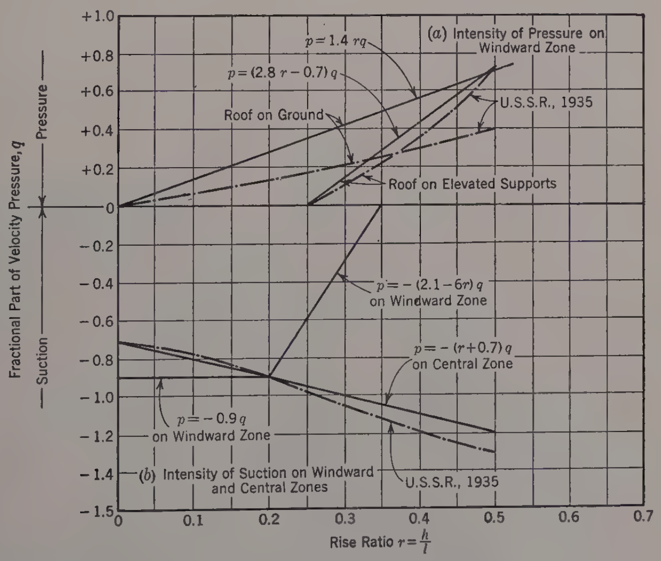

FIG. 3.—RELATION OF WIND FORCE ON ROUNDED ROOF, TO RISE RATIO.

ward zone of roofs having values of r less than 0.20 is more severe than that specified by the latter regulations, which, incidentally, do not provide for a gradual transformation from suction to pressure as the rise ratio of the roof increases. The suction of $0.5\,q$ prescribed for the leeward quarter is more severe than the Russian prescription of $0.4\,q$.

In Fig. 1, the suggested loadings are indicated by heavy lines for a few characteristic rise ratios and for the contrasting cases of roof support at an elevation (full lines) and at ground level (dotted lines).

The nature of the observed external wind force on symmetrical gable roofs is shown in Fig. 2. It is apparent that although for the windward half, the force tends to be suction for the flatter roofs and pressure for the steeper ones, the external force is suction for the leeward half in all cases.

A fairly consistent relation appears to exist between the roof slope and the external pressure for the windward slope. For a slope of approximately 30° the average pressure may be considered as zero; for flatter slopes, it is negative, or suction, whereas for steeper slopes, it is positive, or pressure.

It is recommended that the following loadings be adopted tentatively for the windward slope:

(1) For slopes of 20°, or less, a suction of $0.7\,q$.

(2) For slopes between 20° and 30°, a suction of,

$$p = (0.07\,\alpha - 2.10)\,q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which α is the roof slope, in degrees.

(3) For slopes between 30° and 60°, a pressure of,

$$p = (0.03\,\alpha - 0.90)\,q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and,

(4) For slopes steeper than 60°, a pressure of $0.90\,q$.

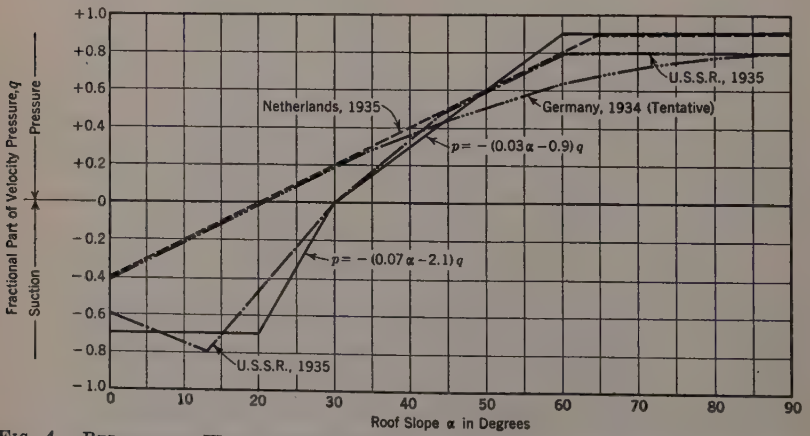

FIG. 4.—RELATION OF WIND FORCE ON WINDWARD SLOPE OF GABLE ROOF, TO ROOF SLOPE.

The proposed units are compared in Fig. 4 with those of the U. S. S. R. Standards Committee, with those of the Main Committee for Standardization in The Netherlands[21], and with those of the Tentative Specification for Wind Loads on Buildings of the German Standards Committee.[22] The suggested loadings are shown to agree very well with the Russian regulations, but to be considerably more severe than those of The Netherlands, or of the German Standards Committee, for small roof slopes. Fig. 2 appears to justify this greater severity.

No consistent relation between the external wind force and slope appears to exist for the leeward half of a gable roof. For the present it is suggested that the force may be taken as a suction of 0.6 times the velocity pressure for all slopes. This appears to be justified from the plotted results in Fig. 2, although both the Russian and the Dutch regulations set it at only 0.4 q.

Fixing of the actual net wind force for design purposes, of course, can only be done by taking into account the combination of external and internal pressures. On the one hand, the internal pressure may be close to atmospheric, due to the fact that there are no appreciable openings for the entry of the outside air. In this case the forces indicated in Figs. 1 and 2 might be used for design. On the other hand, the number, size, and position of open skylights, monitors, windows, or doors may modify considerably the internal pressure. In general, therefore, it will be necessary for the designer to consider an alternative loading, arrived at by superimposing on the external pressures and suctions of Figs. 1 and 2 such difference between the actual internal pressure and atmospheric pressure as the nature of the openings in the structure may create.

For the Irminger and Nokkentved models, having roofs of the gable type, the internal pressures varied from $-0.325\ q$ to $-0.390\ q$ when the wind acted at 90° to the side-walls. These pressures were such as might reasonably be expected in a building having the ordinary degree of air leakage to be found in practice. For Eiffel's Belfort model, with open monitor, the internal pressure was about 0.8 times the velocity pressure below atmospheric.

In the summary of the report on the Russian tests,[9] the authors observe that the internal pressure for buildings of uniform porosity is about $-0.25\ q$, but they point out that for large openings it may be as much as $\pm\ 0.6\ q$, depending on the wind direction with relation to the opening. In the Dutch regulations, provision is made, in the case of a closed building, for an internal pressure of $-0.4\ q$, for one of $+0.6\ q$ when an open side of a building is to windward, and for one of $-0.3\ q$ when it is to leeward. In experimental studies of building models Messrs. E. B. Richardson and B. H. Miller found that, with the wind normal to the side-walls, and when the windows constitute 10% of the wall area, the internal pressure may vary from $+0.8\ q$ to $-0.5\ q$, depending on the combinations of window openings.[23]

Irregularities in building outline or arrangements, of course, will need to be taken into account when fixing both internal and external pressures.

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[21] "Technical Bases for Building Codes—Wind Loads and Concluding Regulations," N790, 1935.

[22] *Der Bauingenieur*, October 12, 1934.

[23] *Journal*, Inst. of Engrs. of Australia, Vol. 4, No. 8, August, 1932, pp. 277–282.

(*C*) ALLOCATION OF WIND FORCE TO WINDWARD AND LEEWARD WALLS

The external effect of wind on the vertical walls of models of buildings is indicated in Table 1. The force coefficients, C, are defined algebraically by the formula,

$$C = \frac{F}{A\,q} \quad\text{..........................}(9)$$

in which F = the wind force on a projected area, A, of the surface considered.

Examination of Table 1 shows that the average external wind force on vertical windward walls is a pressure varying from 0.25 q to 0.88 q, depending on the type of model and the direction of the wind. For the leeward wall

TABLE 1.—FORCE COEFFICIENTS, C, FOR THE EXTERNAL EFFECT OF WIND ON THE VERTICAL WALLS OF BUILDING MODELS

Authority*	Type of model	Ratios of:			Direction of Wind		Average Value of C		C_w+C_l	Floor No.
		Length to width	Eave height to roof span	Roof rise to span	In degrees	Width relation to the:	For windward wall, C_w	For leeward wall, C_l		
Dryden and Hill (11)...........	Mill building....	2.00	0.25	0.18	90	Side wall...	+0.50	−0.52	1.02	....
		2.00	0.25	0.18	90	End wall...	+0.61	−0.21	0.82	....
Dryden and Hill (11)..........	Empire State Building......	1.39	.....	....	90	Side wall...	+0.88	−0.57	1.45	36th
					90	Side wall...	+0.88	−0.72	1.60	55th
					90	Side wall...	+0.80	−0.79	1.59	75th
					90	End wall...	+0.81	−0.56	1.37	36th
					90	End wall...	+0.87	−0.60	1.47	55th
					90	End wall...	+0.78	−0.75	1.53	75th
Eiffel (12).......	Belfort Hangar..	3.33	0.40	0.41	90	Side wall...	+0.61	−0.46	1.07	....
					60	Side wall...	+0.54	−0.50	1.04	....
Irminger and Nokkentved 1	Gable roof......	2.00	1.00	0.18	90	Side wall...	+0.57	−0.31	0.88	....
				0.18	75	Side wall...	+0.64	−0.30	0.94	....
				0.18	60	Side wall...	+0.52	−0.43	0.95	....
				0.18	45	Side wall...	+0.36	−0.45	0.81	....
				0.29	90	Side wall...	+0.57	−0.43	1.00	....
				0.29	75	Side wall...	+0.54	−0.44	0.98	....
				0.29	60	Side wall...	+0.50	−0.45	0.95	....
				0.29	45	Side wall...	+0.35	−0.55	0.90	....
				0.50	90	Side wall...	+0.61	−0.51	1.12	....
				0.50	75	Side wall...	+0.61	−0.51	1.12	....
				0.50	60	Side wall...	+0.55	−0.47	1.02	....
				0.50	45	Side wall...	+0.40	−0.53	0.93	....
				0.87	90	Side wall...	+0.66	−0.54	1.20	....
				0.87	75	Side wall...	+0.62	−0.50	1.12	....
				0.87	60	Side wall...	+0.57	−0.46	1.03	....
				0.87	45	Side wall...	+0.40	−0.55	0.95	....
Smith (17).......	Gable roof......	1.67	1.00	0.20	90	Side wall...	+0.75	−0.14	0.89	....
		1.67	1.00	0.25	90	Side wall...	+0.76	−0.19	0.95	....
		1.67	1.00	0.33	90	Side wall...	+0.66	−0.32	0.98	....
		2.50	1.00	0.20	90	Side wall...	+0.66	−0.22	0.88	....
		2.50	1.00	0.25	90	Side wall...	+0.57	−0.32	0.89	....
		2.50	1.00	0.33	90	Side wall...	+0.58	−0.43	1.01	....
Smith (17).......	Semi-circular roof	1.67	0.25	0.50	90	Side wall...	+0.61	−0.15	0.76	....
			0.33	0.50	90	Side wall...	+0.33	−0.50	0.83	....
			0.42	0.50	90	Side wall...	+0.25	−0.85	1.10	....
			0.50	0.50	90	Side wall...	+0.88	−0.38	1.26	....
			0.58	0.50	90	Side wall...	+0.87	−0.37	1.24	....
			0.67	0.50	90	Side wall...	+0.75	−0.36	1.11	....
			0.75	0.50	90	Side wall...	+0.53	−0.46	0.99	....
			0.83	0.50	90	Side wall...	+0.84	−0.29	1.13	....

*Numbers in parentheses refer to footnotes in the text.

it is a suction varying from 0.14 q to 0.85 q. The arithmetic sum of the pressure on the windward wall and the suction on the leeward wall varies from 0.78 q to 1.60 q, the highest totals being for the model of the Empire State Building, in New York, N. Y. For buildings of moderate height, therefore, the external pressure on the windward wall may be safely taken at 0.9 q and the external suction on the leeward wall at 0.4 q. These are the units prescribed in the Dutch standards. The Russian regulations place the pressure on the windward wall and suction on the leeward wall at 0.8 q and 0.4 q, respectively.

As was stated in the First Progress Report,[24] it is not considered necessary to divide the total wind force on the walls of a typical tall building into pressure and suction effects, although such distribution is desirable for mill buildings, hangars, armories, drill sheds, or similar buildings with large open interiors and for walls in which large openings may occur. To care for especially intense local effects it is suggested that for parts of walls having areas up to 160 sq ft, provision be made for an external pressure of 1.2 q.

(*D*) Simplified Methods of Determining Wind Moments

(1).—Goldberg Method.—In the past two years much use has been made of the procedure for determining wind stresses by slope deflections with converging approximations proposed by John E. Goldberg, Jun. Am. Soc. C. E.[25] and based on the method developed by G. A. Maney, M. Am. Soc. C. E.[26] The results obtained have indicated that the method is one of great merit and that at least in certain commonly occurring types of wind-stress problems, it possesses important advantages over some of the other available methods. An investigation of the respective merits of this method and the Cross method of moment distribution made in the Department of Civil Engineering of the University of Toronto, at Toronto, Ont., Canada, in 1933–34, yielded interesting comparative data, a review of which may be helpful in establishing a basis for judgment.

A bent consisting of three equal bays, and eight stories in height, designed for gravity loads only, was analyzed by a computer, to whom the method was new, with sufficient accuracy in two complete cycles of the Goldberg procedure in 6½ hr. Utilizing moment distribution, three cycles gave satisfactory results in 6 hr. The calculations of deflection due to column flexure required only an additional 10 min by the Goldberg method, whereas, using the moments obtained by the Cross method, 1½ hr. were required. Although, for a low symmetrical bent, there does not appear to be any particular advantage in the Goldberg method, so far as the computation of moments is concerned, there undoubtedly is an advantage in it for finding the deflections. Definite superiority of the Goldberg procedure appeared, however, when the columns of the 8-story bent were designed for the gravity loads pertaining to an additional twelve stories. Analysis by the Goldberg method gave generally satisfactory results with two cycles, requiring 5 hr, the maximum error being about 6% in the bottom story. Moment distribution required six cycles, consuming 7¾ hr, with the same maximum error.

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[24] *Civil Engineering* March 1931, p. 481.

[25] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 962.

[26] *Engineering News-Record*, November 12, 1931; also, *Bulletin No.* 7, Northwestern Univ., Evanston, Ill, 1932.

When the bottom eight stories of a 20-story bent designed for gravity loads and for wind by the Spurr method were analyzed by the Goldberg procedure, three cycles, occupying 7 hr, were necessary in order to reach a convergence of 1% between consecutive results in column moments. Five cycles of moment distribution, taking 9 hr, were required in order to reach a convergence of 3%, in column moments. The 20-story, Wilson and Maney, symmetrical 3-bay bent designed for cantilever action, when analyzed by the Goldberg method required 10¼ hr for one cycle, with a maximum error of 5.7% and 13¼ hr for two cycles, with a maximum error of 4.1 per cent. Five cycles of moment distribution required 15 hr, with a maximum error of 10 per cent. An accuracy as great as the latter was secured by calculation of the moments from the initial estimates of angles by the Goldberg procedure in 6¼ hr.

It is interesting to note that the time required by Mr. Goldberg, personally in calculating the stresses and deflections in the Wilson and Maney bent by his own method was about 6¼ hr, as compared with 13⅓ hr for calculating the moments alone by the method of moment distribution. The time required by the second method conforms fairly closely to that expended in the University of Toronto calculations, but, as might be expected, Mr. Goldberg's special familiarity with his own method enabled him to widen the difference in time between that required for it and that required for moment distribution.

The bottom eight stories of a 20-story, 3-bay, unsymmetrical bent designed by the Spurr method were analyzed by the two methods under consideration, in order to test the effect of asymmetry on these solutions. The Goldberg method required three cycles and 10 hr for a final change of 1%, or less, in the column moments. Moment distribution required five cycles and 18 hr for a final change of 3% in the column moments. The notable saving in time by the Goldberg method in the analysis of this type of bent is easily explainable. By moment distribution the work varies directly as the number of joints to be considered. In solving a slope-deflection problem by converging approximations, however, much of the work is dependent only on the number of stories and in the calculations of twist angles the work is not prohibitively increased by asymmetry. Moreover, the length of the solution is affected very little by the relative stiffnesses of members.

A valuable characteristic of the Goldberg method is that it is admirably suited to the checking of designs of frames having irregularities, where relative joint translation is not a factor. Thus, in the study of the behavior of the model of a 55-story, 3-bay bent conducted recently at the Engineering Experiment Station of the Ohio State University, at Columbus, Ohio, the method was found decidedly superior to all other well-known methods, including moment distribution and its variations, for the checking of the design of the lower part of the prototype bent with a 2-story entrance. It was established that for reasonably well-designed bents the approximate or estimated values of the joint rotation angles were very close to the final values obtained after following through three or four cycles. Girder shears calculated from these approximate values were within 5% of those obtained by a rigorous slope deflection solution.

Although the Goldberg procedure may be applied to the determination of secondary moments, it is not so simple in application as the method of moment distribution. By either method the fixed-end moments due to column length changes must first be computed on the basis of the offset of one end of each girder with respect to the other. Following the Goldberg method, the moments thus found must be used to calculate joint rotations and story deflection angles, and these angles must be re-translated into moments. Obviously, the procedure is too roundabout to be expeditious.

(2).—Grinter Method.—A simplified method of analysis based on the Cross procedure of balancing fixed-end moments developed by Professor Grinter,[27] makes possible for frames with very stiff columns a saving in the time of solution as compared with that necessary when following the ordinary Cross procedure with successive shear corrections.[28] By making a fair estimate of deflection at the outset, and correcting moments by proportion at the finish, one may achieve a satisfactory result without performing a large number of cycles of moment distribution. Fortunately, except in highly irregular frames, the deflections arising in each story may be estimated within a variation of 10%, which will give final moments with a maximum error of about 6 per cent.

The times required for the analysis of a frame by the Grinter and the Goldberg methods do not appear to differ greatly. Personal preference will probably have much to do with the choice. The former method would seem to give the computer a little clearer conception of the physical behavior of the bent.

Apparently, secondary moments can be determined somewhat more readily by the Grinter method than by the Goldberg method. The importance of these moments, however, has probably been over-estimated, and in very many cases they need not be computed at all.

(*E*) Experimental Studies of the Behavior of Wind Bents

In developing satisfactory methods of design and analysis of wind bents much aid may be derived from study of the behavior of models. Two recent investigations having an important bearing on the work of the Sub-Committee are the tests of model wind frames (1) at the University of Pennsylvania, at Philadelphia, Pa., and (2) at the Ohio State University.

(1).—Tests at the University of Pennsylvania.—A description of the construction and use of a model which has been employed for some time in the Civil Engineering Department of the University of Pennsylvania, together with an account of the results obtained in the study, has been presented by Francis P. Witmer, M. Am. Soc. C. E., and Mr. Harry H. Bonner.[29] A

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[27] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 610.

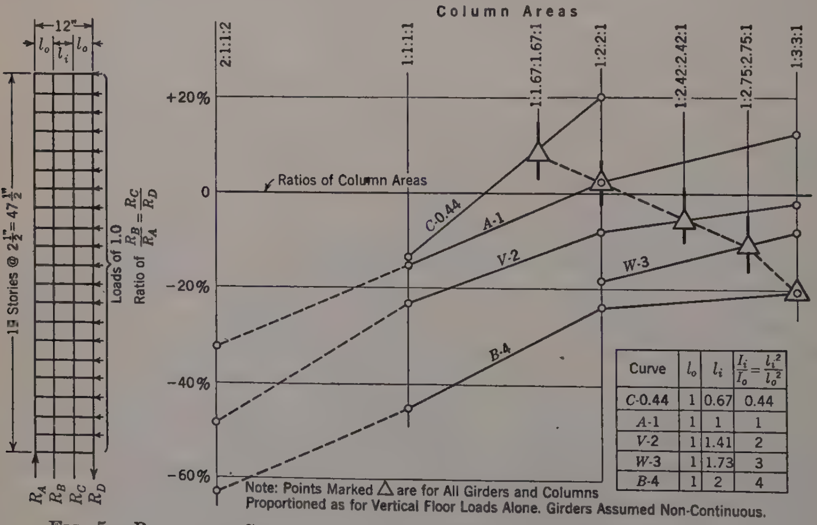
[28] *Proceedings*, Am. Soc. C. E., February, 1932, p. 224.

[29] "Tall Building Frames Studied by Means of Mechanical Models," by Francis P. Witmer and Harry H. Bonner, *Proceedings*, Am. Soc. C. E., January, 1936, p. 3.

principal portion of this study was with reference to symmetrical 4-column bents. The relative width of bays, relative moment of inertia of girders, and relative size of columns were subjected to wide variation, and experimental determination was obtained of the ratio of vertical reactions at the foot of inner and outer columns resulting from the application of transverse forces at the different floors.

The results of the more than twenty cases tested showed that the natural tendency of bents, with girders designed in the proportions required for vertical floor loads only, was more nearly in agreement with the portal than with the cantilever theory, but that any desired ratio of vertical reactions could be produced by properly adjusting the moments of inertia, *I*, of the girders, and, to a less degree, by varying the relative size of columns. To obtain the cantilever relation of reactions, an increase in the moment of inertia of center girders over that required for vertical floor loads was always required, amounting in some cases to a very large percentage and being least for bents with a narrow center bay and greatest for those with a wide center bay.

Subsequent investigation has been made covering symmetrical 4-column bents of varying relations of bays, all having a height of 19 stories and a height-to-width ratio of 4. All parts were designed with the proportions required for vertical floor loads only, girder moments of inertia varying as the square of their span length, and column areas varying as the reactions for non-continuous girders. Furthermore, in all bents, the size of the minimum column was made the same as that of the minimum girder, and each column had a constant size from top to bottom.



Curve	l_o	l_i	$\frac{I_i}{I_o} = \frac{l_i^2}{l_o^2}$
C-0.44	1	0.67	0.44
A-1	1	1	1
V-2	1	1.41	2
W-3	1	1.73	3
B-4	1	2	4

Note: Points Marked △ are for All Girders and Columns Proportioned as for Vertical Floor Loads Alone. Girders Assumed Non-Continuous.

FIG. 5.—RATIOS OF COLUMN REACTIONS FOR VARIOUS BENT PROPORTIONS; TESTS AT THE UNIVERSITY OF PENNSYLVANIA.

Fig. 5 shows curves for five different bay ratios with girders designed for vertical floor loads. Column sizes were varied for each case and column area ratios were plotted as abscissas, while column reaction ratios were plotted as ordinates. The points actually tested are indicated by circles. The column-area ratio corresponding to vertical floor loads for each of the five cases is shown by a heavy vertical line and its intersection with the corresponding curve, shown by a triangle, indicates the reaction ratio for that particular case. Within the range of bay ratios studied, the following conclusions are indicated: If girders and columns are proportioned for vertical floor loads only, as described herein: (*a*) Reactions for inner columns will be practically zero for equal bays; (*b*) reactions for inner columns will be less than + 10% of those for outer columns if the center bay is narrower than the outer bays; and (*c*) reactions for inner columns will be less than — 10% of those for outer columns if the center bay is not more than one and one-half times the width of the outer bays.

Further study is planned to discover the effect of changing the size of columns relative to girders, but maintaining always the same relation between the columns of a bent, in order to preserve the vertical load relation of areas. It is also planned to study the effect of varying the size of columns in different stories. This additional information is essential, of course, in order that the results of the research may be applied to actual building designs. The present intention is to point a way of adapting the design of such bents to their actual behavior as determined experimentally, and thus to avoid unnecessary sacrifice of economy in material.

(2).—Tests at Ohio State University.—Early in 1934 a steel model of the lower thirteen stories of a Spurr-designed 55-story, 3-panel symmetrical bent was constructed and tested at the Ohio State University, by Clyde T. Morris, M. Am. Soc. C. E., George E. Large, Assoc. M. Am. Soc. C. E., and Samuel T. Carpenter, Jun. Am. Soc. C. E. The model was necessarily three-dimensional, the columns being H-sections built up by spot-welding Since only members were being studied, perfect connections were achieved by cutting the girder system from one continuous sheet. The model bent was tested lying flat upon steel tables with the base anchored to a concrete pier, the wind loads being applied by dead weights operating over pulleys[30].

The results of the strain-gage and deflection data taken verified the Spurr method in so far as the routine designing of regular and symmetrical bents is concerned. As published, it was found quite inadequate for designing in the vicinity of structural irregularities, such as fixed column bases, tall basement stories, and 2-story entrances.

A tall basement story was then created by removing the first floor girders from the model, and re-testing. This change resulted in the lower stories deflecting substantially as designed, since points of contraflexure in the basement story occurred at about three-quarters of the distance up from the fixed base, or 6 ft below the lowest girder. As a result of such studies the investigators have developed a procedure[31] for designing the several stories immediately above, and affected by, column bases of full or partial fixity. Near such points the investigators wish to emphasize that the designer must keep fundamentals clearly in mind and then check his design, preferably by the Goldberg[25] approximate slope-deflection procedure, since it has been found admirably suitable for checking designs involving several irregular stories, especially where the columns are relatively very stiff compared to the girders.

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[30] "Tests of a Steel Model of a 55-Story Wind Bent"—A Progress Report, November, 1934, pub. by the Am. Inst. of Steel Construction.

[31] "Tests and Design of Steel Wind Bent Irregularities"—Second Progress Report of Model Investigation, June, 1935, pub. by the Am. Inst. of Steel Construction.

Test data taken about a 2-story entrance created later indicated a maximum increase in girder stress of about 19% in the girders on each side of the entrance at its half height, while the girder directly above the entrance suffered an increase of only 12 per cent. All points of contraflexure in the girders, remained practically at the center.

The set-back is another structural irregularity that has been studied thoroughly in a theoretical manner, since a diligent search of engineering literature revealed very little information of value on this point. For full panel set-backs, the "increment" method of considering the story increment of wind overturning moment taken by the outside columns below the set-back floor was the only practicable stress assignment found. This method results in equal girder shears across a floor, with only the outside columns of the base off the straight-line variation of direct unit stress. Beginning at the ground the accumulated direct deformation may be computed and the deviation allowed for in selecting the girders of the outside panels. A set-back model so designed is being constructed for test. It will have a six-column base with a very wide (28-ft) center panel.

The complicated case of several columns being carried over banking rooms or assembly halls on heavy trusses has also been studied by Mr. Carpenter who is constructing a steel test model involving this type of irregularity at Swarthmore College, Swarthmore, Pa., under a co-operative agreement with the Ohio State University Engineering Experiment Station. In addition, a three-panel unsymmetrical bent is being designed for model test by Mr. Leon Seligson.

The Sub-Committee wishes to express its special thanks to Dr. H. L. Dryden, U. S. Bureau of Standards, and to Lieut. H. MacT. Sylvester, C. E. C., U. S. N., Jun. Am. Soc. C. E., for assistance received in connection with the study of the character and magnitude of the wind force.

Respectfully submitted,

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December 28, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

LINE LOAD ACTION ON THIN CYLINDRICAL SHELLS

Discussion

BY HERMAN SCHORER, ASSOC. M. AM. SOC. C. E.

HERMAN SCHORER,[27] ASSOC. M. AM. SOC. C. E. (by letter).[27a]—An interesting outline for the analysis of a sector gate is contained in the discussion by Mr. Silverman. In steel structures of this type, with comparatively heavy surface loads, it may become necessary to make provisions against buckling of the skin-plate in a longitudinal and radial direction, depending on the relative dimensions. As a rule, the shell is comparatively stiff in the longitudinal direction and the rim-bending stresses are limited to a narrow zone along the stiff end members. For this reason the intermediate longitudinal members, as shown in Fig. 9, could probably be eliminated. However, the danger of buckling in a radial direction would require closer investigation since the resistance of the arch is reduced by the large radial bending moments, originating from the heavy longitudinal edge member at $\phi = 0$. The necessary stiffness in a radial direction can be obtained by the provision of small radial ribs, welded to the skin-plate, perhaps combined with a few longitudinal ribs at the center zone of long spans.

Dr. Flügge's solution for J_2 and K_2 in explicit form represents a desirable improvement and will be welcomed by those who are interested in a more exact analysis of the problem. Mr. Tedesko contributes a practical illustration of the stress components in the roof of the Market Hall, in Budapest, one of the most outstanding shell structures.[27b]

The writer is greatly indebted to Dr. Finsterwalder for pointing out the exact solution of the entire problem, as recently published by Professor

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NOTE.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in March, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1935, by I. K. Silverman, Jun. Am. Soc. C. E.; November, 1935, by Messrs W. Flügge, and Anton Tedesko; and December, 1935, by Messrs. U. Finsterwalder, and F. W. Seidensticker.

[27] Gen, Mgr., Borsari Tank Corporation of America, New York, N. Y.

[27a] Received by the Secretary January 28, 1936.

[27b] In Figs. 11(*b*), 11(*c*), 11(*d*), and 11(*e*), of this discussion the solid line curves denote values determined by the accurate theory (considering the first two terms of the Fourier Series); and the broken line curves denote values determined for the membranous condition.

Dischinger[24], which enables the designer to include the stress components and simplifications neglected thus far. It also includes anistropic shells (that is, shells with non-uniform bending resistance in different directions), such as those introduced by a series of radial or longitudinal stiffening ribs.

The discussion by Mr. Seidensticker is encouraging and his reference to the full-sized loading tests in Chicago is very timely. The excellent behavior of these structures under non-symmetrical loadings appears in striking contrast to the expectations derived from pure arch action. In conclusion, the writer wishes to express his sincere thanks to all those who have discussed the paper.

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[24] "Die strenge Theorie der Kreiszylinderschale in ihrer Anwending auf die Zeiss-Dywidag-Schallen", von Prof. Dr. F. Dischinger, *Beton und Eisen*, Vol. 34, August, 1935, *et seq.*

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PHOTO-ELASTIC DETERMINATION OF SHRINKAGE STRESSES

Discussion

BY ARSHAG G. SOLAKIAN, ESQ.

ARSHAG G. SOLAKIAN,[12] ESQ. (by letter).[12a]—The experimental method used by Mr. Smits, namely, of stretching the base of the model to introduce shrinkage stresses in the body of an "idealized" dam, does not well represent the actual conditions prevailing in Nature as will be demonstrated presently.

Shrinkage stresses in concrete dams of the gravity type are produced under several conditions, the most important of which are: (1) The setting of the cement in concrete, in a dam resting on a clay foundation or loose rock; (2) the setting of the cement in concrete, in a dam having a perfect bond with a solid rock foundation (Mr. Smits' paper deals only with this case); and (3) the daily (or weekly) and seasonal changes of atmospheric variations of temperature, in a dam having a perfect bond with a solid rock foundation.

Condition (1).—Stresses in this catagory are due to the premature setting of the cement at the two exposed surfaces of the dam, before such action begins to develop in the interior regions of the cross-section. As a result, a relatively harder crust is formed around the structure in Nature's attempt to resist subsequent shrinkage effects of the interior region. Such a phenomenon and the resulting stress distribution will be similar in Nature to that which occurs in every day foundry practice when dealing with cast metals.

Stresses of the intensity and direction produced under Condition (1) can be obtained by photo-elastic methods from a model cast under proper curing conditions. In a previous investigation[13] bakelite specimens revealed the existence of such stresses with a sufficient number of interference fringes. A model was prepared by filling with bakelite powder a metal form 1 in. deep, having the outlines of the profile of the dam with extended footings. The

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NOTE.—The paper by Howard G. Smits, Jun. Am. Soc. C. E., was published in May, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: September, 1935, by Messrs. Thomas H. Evans, and I. K. Silverman; and October, 1935, by J. H. A. Brahtz, Esq.

[12] Lecturer in Civ. Eng., Columbia Univ., New York, N. Y.

[12a] Received by the Secretary December 6, 1935.

[13] "Photo-Elastic Analysis of Stresses in Composite Materials", by A. H. Beyer, M. Am. Soc. C. E., and A. G. Solakian, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p 1196.

material was heated for several days at a constant temperature and then gradually cooled to room temperature. After being machined to a thickness of about $\frac{5}{8}$ in. for plane parallel surfaces, the cast plate was examined in circularly polarized light for the fringe pattern in Fig. 13. In this view each fringe line is a contour of the constant difference between the principal stresses, σ_1 and σ_2 acting at a point, with a relative intensity for each fringe line equal to its order as marked. Since the maximum shear stress intensity is known to be equal to one-half the difference of the principal stresses, the fringe pattern will also show the distribution of these types of stresses which is quite useful for actual design purposes under certain circumstances. The variation of these stresses across three typical sections, *A–A*, *B–B*, and *C–C* (Fig. 13)—coinciding, respectively, to the base, one-third, and two-thirds the height of the dam—are shown in Fig. 14(*a*). From Figs. 13 and 14(*a*) it is evident that sharp corners and fillets of small radii are regions of high stress concentrations under the conditions of structure as discussed (Condition (1)). Furthermore, the stress gradient increases from the center of the dam toward the outer surfaces, for all sections considered.

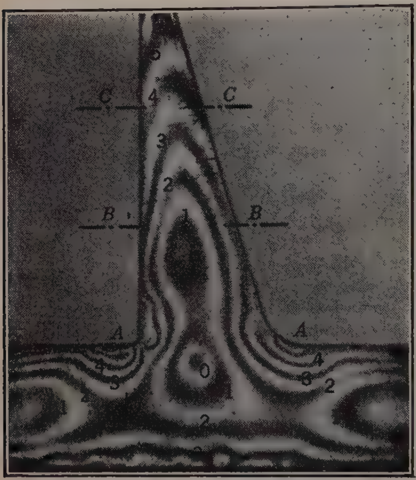
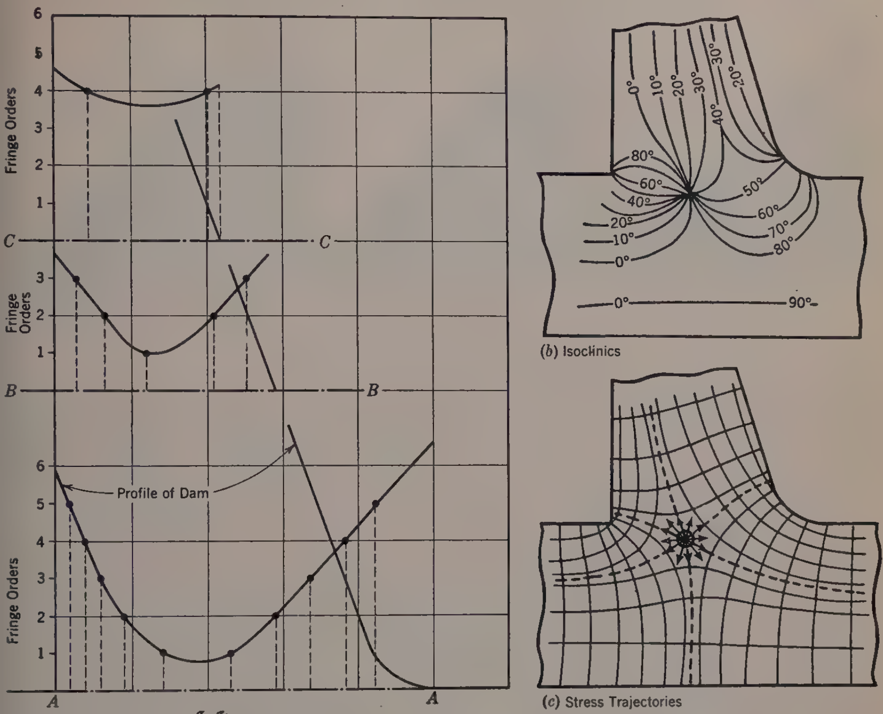


FIG. 13.—FRINGE PATTERN.

The orientation of the principal stresses, σ_1 and σ_2, is found by using plane polarized light at constant directions of vibration from 0° to 90°, with 10° intervals. The resulting isoclinic bands that are contours of constant orientation of these stresses are traced as in Fig. 14(*b*). From these curves, the stress trajectories of Fig. 14(*c*) are obtained by a graphical process. The existence of a singular point (known also as an isotropic or neutral point) near the base of the dam is clearly evident from Figs. 13, 14(*b*), and 14(*c*). At such a point the two principal stresses are at all possible orientations, and are of equal magnitude and sign (both tension or both compression). Note, for example, that $\sigma_1 - \sigma_2 = 0$, represents the zero order fringe in Fig. 13.

Condition (2).—When the dam has a perfect bond with an underlying foundation, the method outlined under Condition (1) does not apply. In this case a mass of infinite extent which forms an integral part with the base of the dam, is free from shrinkage phenomena, since it is underground and at a constant temperature. Such action, however, is developing within the entire body of the dam. The method proposed by Mr. Smits serves to introduce stresses only in the base region of the dam, whereas the larger part of the structure above this region is not affected. A logical method would be to make a model consisting of two pieces, a large block to represent the massive foundation and a small triangular plate to represent a thin section of the dam.

The block is kept in a bath of hot water, whereas the thin plate is kept in dry ice, both long enough to establish a uniform temperature distribution throughout their entire mass. The two pieces then would be cemented together along the base of the dam, taking care that each section was kept at its original uniform temperature during the final setting of the cement. The cemented model would then be introduced into a cell with glass walls and containing hot water, to raise the temperature of the dam section to that of its foundation and thus to make it expand, the block as a whole being unaffected. This action is similar to that which occurs under actual conditions in the structure considered. Fig. 15 is a fringe pattern of the stresses produced in a model examined by the method as described. A new photoelastic material with a greater stress-optical sensitivity, was used for the two parts of the cemented model. A special quick-setting cement was also used.[14] An inspection of Fig. 15 makes evident the existence of shrinkage stresses all over the cross-section of the dam, in variance with that of Fig. 5 in the paper.



(*a*) Maximum Shear $\left(\frac{\sigma_1-\sigma_2}{2}\right)$ Stress Distributions

(*b*) Isoclinics

(*c*) Stress Trajectories

FIG. 14.—STRESSES IN A DAM ON CLAY FOUNDATION, DUE TO SETTING OF CEMENT IN THE CONCRETE.

Condition (3).—The effect of temperature variation producing shrinkage stresses in concrete dams can be reproduced by a process similar to that discussed under Condition (2). The model, however, can be made from a single plate if desired. The large base plate is kept in a bath of hot water, whereas the dam section is enclosed in a cell with glass walls. The glass plates are in contact with the polished surfaces of the dam section, and the two compartments thus formed on either side of the dam are filled with dry ice, or the up-stream cell with hot water and the down-stream cell with dry ice, as it is desired to duplicate the cases of the reservoir filled or empty. With this arrangement, flow of heat through the body of the dam will be restricted across the two edge surfaces and the base of the dam.

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[14] "A New Photoelastic Material", by A. G. Solakian, *Mechanical Engineering*, December, 1935.

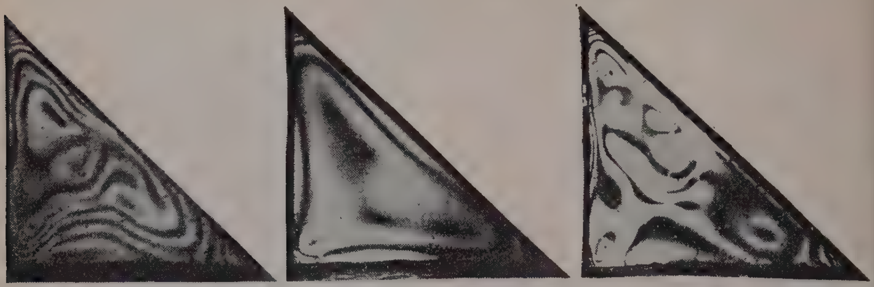
(*a*) DUE TO THE SETTING OF CEMENT.
(*b*) DUE TO DAILY VARIATIONS OF ATMOSPHERIC TEMPERATURE.
(*c*) DUE TO SEASONAL VARIATIONS OF ATMOSPHERIC TEMPERATURE.

FIG. 15.—STRESSES IN A DAM ON ROCK FOUNDATION.

When the effects of a sudden change of temperature (daily or weekly atmospheric variations) are considered, the fringe pattern in Fig. 15, taken a few minutes after the model is put in the baths of hot water and dry ice, will give a qualitative picture of the resulting momentary temperature-stress potential that exists over the cross-section of the dam. To duplicate the effects of seasonal variations of atmospheric temperature, a sufficient time of heat flow should be allowed in the test, to produce a more or less constant flow of heat from one part of the dam section to the other, thus affecting the interior points of the dam as well. Under these conditions, then, the fringe pattern will be quite steady provided the temperature of the source of heat and the sink is kept constant. Fig. 15(*c*) represents the latter situation.

Due to the necessity of certain improvements in the details of conducting the test, as described under Conditions (2) and (3), further analysis with curves showing the intensities and orientations of the stresses for the last two cases is disregarded. As a matter of fact, because of the small size of the models used in this discussion (3-in. legs of dam section) and by Mr. Smits, the experimental results should be considered only from a qualitative point of view.

The suggestions described herein, for the investigation of shrinkage stresses arising from various causes, combined with the centrifuge-polariscope method[15] of analyzing the stresses in dams due to gravity and hydrostatic pressure, should give the investigator sufficient tools for a complete and exact experimental solution of the dam problem.

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[15] "Centrifuge Method of Testing Models", by P. B. Bucky, A. G. Solakian, and L. S. Baldin, *Civil Engineering*, May, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SOME LOW-TEMPERATURE CHARACTERISTICS OF BITUMINOUS PAVING COMPOSITIONS

Discussion

BY MESSRS. M. HIRSCHTHAL, AND HUGH W. SKIDMORE

M. HIRSCHTHAL,[17] M. AM. SOC. C. E. (by letter).[17a]—The question raised by Mr. Skidmore in his analysis of bituminous mixtures as used for paving materials is also of great importance in the use of bitumens for water-proofing purposes where exposed to weather.

Unfortunately, it is a fact that although bitumens have been refined to develop qualities that will prevent "running" in high temperatures, the attempts at prevention of brittleness at low temperatures (at and below freezing) have been far from successful.

If a water-proofing material or a paving material is to be used in a location where it will be subjected to temperatures below freezing, the material should have some ductility at those temperatures to meet the contraction which is occurring and the expansion that is to follow with a future rise in temperature. It has been claimed that ductility tests on bitumens at 40° F are not reliable, as slight jars tend to crack the bitumen at that temperature before the test is completed. The claim itself indicates the necessity of ductility that is measurable. At the same time, it indicates the requirement of improved technique in making the test. Another claim frequently made is that a ductility test at 77° F, together with a melting point and penetration of the bitumen, will indicate the ductility at 40° F without the test at that temperature.

In 1921 and 1922 the writer required tests of quite a number of bitumens by the laboratory of the Delaware, Lackawanna and Western Railroad Company, at Scranton, Pa., for the purpose of obtaining results as a basis for writing a specification for water-proofing for the improvements on that road at East Orange, N. J. (at that time there was no water-proofing specification for railroad structures in existence). Some of the bitumens then tested and

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NOTE.—The paper by Hugh W. Skidmore, Assoc. M. Am. Soc. C. E., was published in August, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs. Philip W. Henry, J. T. L. McNew, and Roy M. Green; and December, 1935, by Messrs. W. W. Crosby, and Joseph Zapata.

[17] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

[17a] Received by the Secretary December 19, 1935.

others submitted later to the laboratory showed melting points, penetrations, and ductility at 77° F, within the range of those found acceptable, but had no ductility at 40° F—a confutation of the claim that the other qualities of the bitumen will be sufficient to indicate a ductility of the material at 40° F. The writer feels strongly the necessity of knowing the quality of ductility at low temperatures, in a material that it is known will be subjected to such temperatures on structures which are subject to movement due to such changes in temperature.

HUGH W. SKIDMORE,[18] ASSOC. M. AM. SOC. C. E. (by letter).[18a]—The reasoning of Professor McNew in connection with the fallacy of using hard asphalt in pavements, is very sound. It coincides perfectly with the ideas of the writer and his associates.

The displacement of paving mixtures under traffic during hot weather is the direct result of shearing action in the structure, induced by an applied force which is mainly compressive in nature. This compressive force is applied by means of the wheels of vehicles passing over the pavement. Fast-moving traffic ruts the pavement longitudinally in the direction of travel by displacing the composition laterally; the tendency, if any, toward forward displacement is converted apparently into lateral displacement due to the speed of the moving wheels. Vehicles coming to a stop on the pavement tend to displace the composition in a forward direction, creating humps across the line of travel. Very heavy, slow-moving traffic causes both types of failure.

In so far as hot-mix, filled, or dense, graded types are concerned, the problem of designing mixtures which will resist displacement successfully, has been readily solved, following the development of stability tests in which shear is involved. In the case of oil aggregate and some of the cold-lay mixtures, shear tests cannot be used at temperatures consistent with service conditions because these compositions possess little resistance to direct shear, except at very low temperatures; and yet many of the very soft and open-graded types of mixtures have shown satisfactory stability against displacement under traffic. This has led to the development of a new type of test as reported by Thomas E. Stanton, Jr., M. Am. Soc. C. E. and Mr. F. N. Hveem.[19]

Research in soil science is throwing much light upon the matter of the stability of cohesive masses subject to plastic flow, as well as upon the mechanics of stability of masses of non-cohesive, granular materials. Bituminous paving mixtures obviously are cohesive and subject to plastic flow. The stabilometer developed by Messrs. Stanton and Hveem appears to show much promise in the determination of the stability of all classes of bituminous paving mixtures as well as soils.

The low-temperature performance of bituminous paving mixtures, however, does not involve stability of the compositions. It has been shown clearly that the characteristics of bituminous cement are highly important with respect to low-temperature performance of the mixture. Cements that remain ductile at low temperatures unquestionably resist cracking. In this connection, Mr. Green points out that California asphalts have generally given good service in cold weather. The writer is entirely familiar with this fact. Although California cements of 50/60 penetration show no ductility at 0° C or 5° C, when elongated at a speed of 5 cm per min, they do show excellent ductility when a rate of $\frac{1}{4}$ cm per min is used; also, they show excellent elongation in the cementing value test, in which case the rate is 1 cm per min, whereas the tar and the highly cracked synthetic asphalt show no ductility at these low temperatures at either slow speed or fast speed.

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[18] Pres., Chicago Paving Laboratory, Chicago, Ill.

[18a] Received by the Secretary January 25, 1936.

[19] *Proceedings*, 14th Annual Meeting, Highway Research Board, Washington, D. C., December 6 and 7, 1934.

Mr. Zapata raises the point of the stability of the bitumen-free aggregate, although he does not describe the nature of the test which he uses for this purpose. Mr. Zapata's test measures the horizontal shear of a confined specimen of either coated or uncoated mineral aggregate, and he finds that the uncoated mineral always shows a higher value than the coated aggregate. Obviously, such a test does not measure stability as stability functions in the pavement or in the sub-grade; neither does it measure direct shear, because the specimen is confined laterally. Possibly, it is a measure of the mechanical interlocking of mineral grains, although it does not simulate service conditions. According to the Zapata test, uncoated, dry sand is more stable than a sand-bitumen or a sand-water mixture, and yet it is well known that dry sand will not support vehicular traffic, whereas wet sand will, as will also sand-asphalt mixtures.

With reference to Mr. Zapata's criticism of the lack of sufficient data concerning the characteristics of the bituminous cements, suffice it to state that the cements used in the studies were subjected to several tests not included in the general run of specifications for paving binder. No doubt others also could have been included as an after-thought. Most certainly the ductility test is one for quality and not for identification of source, as Mr. Zapata infers. The cementing-value test is well known to technologists and is described in detail in the reference given.[16]

Reference to Fig. 4 will show that binders of 44 to 175 penetration were studied. Surely this pretty well covers the range of cements suitable for hot-mix construction. Also, contrary, to Mr. Zapata's statement, the cement showing the highest shear strength at the low temperatures has good ductility at 41° F, even with its high mineral content present. Mineral-free Trinidad cement is unusually ductile at low temperatures unless an inferior flux is used.

There can be little question but that alternate freezing and thawing is destructive to any structure sufficiently porous to admit appreciable quantities of moisture. It is to avoid this difficulty, as well as to increase stability, that highway engineers generally are recommending closer grading and better filling of the aggregates used in low-cost road mixtures. Thermal contraction cracking unquestionably will be more severe at − 20° F than at 32° or 41° F. Bituminous pavements must not be confused with Portland cement concrete in this respect, since concrete slabs are generally high in moisture when cold weather comes, and are probably most susceptible at or near 32° F. The moisture content of a well designed and well built hot-mix asphalt paving mixture is very low, being usually less than 0.5% after total immersion of 3 to 5 days. Bituminous surfaces on concrete bases are compelled to follow the movement of the base irrespective of their own ability to resist contraction forces.

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[16] *Journal of Industrial and Engineering Chemistry*, Vol. 6, No. 12, p. 976.

The writer fails to appreciate Mr. Zapata's analogy to the effect that since the tar mixture showed the highest shear strength at 41° F, the tar binder should have shown the highest ductility at that temperature. There is nothing in the data to indicate remotely that such a conclusion could be drawn. The fact remains demonstrated by these studies that those bitumens which showed the best ductilities at 41° F also produced the best shear strengths throughout the low temperature range, except the Bermudez cement, which reversed its curve in the sub-zero range.

The exact composition of the two mixtures which Mr. Zapata selected was:

	Specimen 5	Specimen 10
Bitumen (pure), by weight.......	10.6%	9.7%
Filler, by weight................	17.9%	18.1%
Sand, by weight..................	71.5%	72.2%

In every case adjustment was made to accommodate differences in specific gravity and purity of cement. Actual proportioning was by volume although the formula is based on weight.

It is not clear what Mr. Zapata means by "the durability of the asphalt cements." It was the writer's purpose in presenting the results of the study to try and throw additional light upon the matter of durability of bituminous pavements especially with respect to low temperatures.

It is quite possible that the ductility test will be supplanted by a better tool. Considerable can be said for the cementing value test. Neither of these tests seems to give all the information desired, but it must be remembered that several tests are required to establish the quality of a bituminous material and that some of the tests now being made are worse than useless since they may give fictitious information. A particularly culpable test which falls in the latter category is the reduction test at 500° F, made upon liquid binders to provide a so-called asphaltic residue of 100 penetration at 77° F.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FLOOD AND EROSION CONTROL PROBLEMS AND THEIR SOLUTION

Discussion

BY E. I. KOTOK, ESQ., AND C. J. KRAEBEL, ESQ.

E. I. KOTOK,[23] ESQ., AND C. J. KRAEBEL,[24] ESQ. (by letter).[24a]—A great volume of data on rainfall and run-off relations in Los Angeles County, California, has been accumulated by the County Flood Control District and various Federal agencies. A smaller amount of information on erosion has been similarly gathered. Mr. Eaton has done a signal service in selecting some of the most significant facts and estimates and presenting them with his excellent analyses. The engineering methods described for flood and débris control have not only proved successful where they have been used, but are indicated as indispensable for the protection of communities contiguous to mountain water-sheds which have been denuded of vegetation.

In stating that the natural cover of vegetation affords the "best protection from débris", Mr. Eaton expresses a fact which has been so frequently and spectacularly demonstrated in California by the sequence of fire and flood that it is now quite generally accepted. The formulation of adequate plans for water-shed management, however, still requires the obtaining of quantitative data on the degree of influence exerted by different types and conditions of forest cover, not only upon soil movement, but also upon water movement. Such planning requires, furthermore, a study of the effects of manipulating or "improving" the cover by cutting, planting, or otherwise, and the development of methods of restoring vegetative control quickly after accidental denudation.

In these problems the forester's interest is vital, since in California nearly all the major water-sheds lie within National forests. In Southern California

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NOTE.—The paper by E. Courtland Eaton, M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs. Arthur G. Pickett, and R. W. Davenport; December, 1935, by C. S. Jarvis, M. Am. Soc. C. E.; and February, 1935, by Messrs. Harry F. Blaney, W. P. Rowe, and J. B. Lippincott.

[23] Director, California Forest and Range Experiment Station, U. S. Forest Service, Berkeley, Calif.

[24] Senior Silviculturist, California Forest and Range Experiment Station, in Charge of Studies in Water-Shed Management and Erosion Control, U. S. Forest Service, Berkeley, Calif.

[24a] Received by the Secretary January 23, 1936.

where water is the limiting factor of human development, the primary objective of forest management is to produce from mountain catchment areas the maximum annual yield of useful water—with emphasis on the word "useful" and with due regard for the increasingly important secondary use of the forest in this region, namely, recreation.

The California Forest and Range Experiment Station, representing the research branch of the U. S. Forest Service, is engaged in a comprehensive program of studies in water-shed management[25], including both field and laboratory attacks and giving first priority to the aforementioned problems. A large part of this work is centered in the San Dimas Experimental Forest, an area of 17 000 acres within the Angeles National Forest, set aside for research purposes by special order of the Chief Forester, U. S. Forest Service. The area embraces the entire water-sheds of Big Dalton Creek (7.5 sq miles) and San Dimas Creek (18.3 sq miles), situated in the San Gabriel River drainage and shown in Fig. 1, both commanded near their canyon mouths by large dams built in 1928 and 1922, respectively, by the Los Angeles County Flood Control District for flood control and water conservation purposes. During the three years, 1933, 1934, and 1935, with the help of unemployment relief labor, the Experimental Forest has been equipped with approximately 350 rain-gauges, 17 stream-gauging stations, 9 débris measuring reservoirs, numerous climatic stations, sample plots, and the necessary roads, trails, telephone lines, laboratory, and other service buildings. All engineering design and construction has been done under the direction of the Regional Engineer of the Forest Service, and at all stages, the staff has invited criticism and suggestions from local engineers, water officials, and other interested persons, in the effort to make of the Experimental Forest an adequate laboratory of water-shed management and erosion control problems for Southern California.

In discussing the 1934 flood flow from Pickens Canyon, the auther touches on the difficulty of calculating such a flow from high-water marks and channel sections after the flood. A characteristic of such flows is their scouring action by which the stream beds may be lowered several feet during a single storm. Both the exposed bed-rock of the scoured channels and the roughly stratified structure of the great outwash fans at their mouths constitute evidence that such scouring has occurred periodically through past centuries. A survey of Pickens Canyon made by members of the Experiment Station after the flood revealed soil loss from $\frac{1}{2}$ mile of the canyon bottom at the rate of 20 000 cu yd per lin mile of channel. Fig. 15 depicts a typical section of the channel, photographed after the Montrose flood of January 1, 1934, which illustrates the difficulty encountered in estimating run-off rates of débris-laden flows. The upper tape, *A*, indicates the highest water stage, the middle tape, *B*, the pre-storm channel bed, and the lowest tape, *C*, the present scoured channel as it was left by the flood. Erosion of the stream bottom was progressive. The question, for which there is no definite answer, is: "What was the channel profile when the water surface was at the highest level?".

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[25] The program is described briefly by Mr. E. I. Kotok, and some representative data are presented by Mr. C. J. Kraebel, in the Report on Progress Conference on Water Conservation, Los Angeles, Calif., March 13–14, 1935, by the Committee on the Conservation of Water, Irrigation Division, Am. Soc. C. E. (mimeographed December, 1935).

It is quite possible for the maximum run-off to have occurred when the channel was only partly scoured and the water surface had dropped below its highest mark.

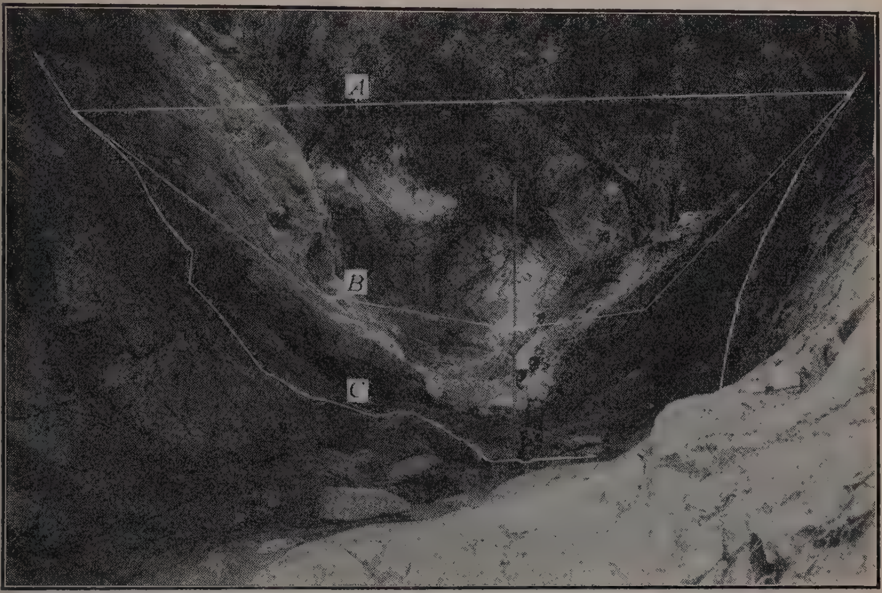

FIG. 15.—CHANNEL OF PICKENS CANYON, AFTER MONTROSE FLOOD OF JANUARY 1, 1934.

In the effort to find a positive means of measuring débris-laden stream flows, the staff of the Experimental Forest Station has found the Parshall flume unsatisfactory and is developing a critical-depth flume with parallel sides and sloping floor. Tests of this new meter have thus far been very encouraging.[26]

Regarding estimates of erosion with normal water-shed cover the author rightly states that information is not plentiful. A probable reason for this lack of data is that débris production from well-covered chaparral water-sheds is so slight that it ordinarily escapes notice. The erosion rates given for the San Gabriel, Gibraltar, and Sweetwater water-sheds (see heading "Erosion with Normal Water-Shed Cover") do not represent erosion from well-covered areas because all these water-sheds had been severely burned over, and the Sweetwater water-shed had also been over-grazed for many years. There are so many variables involved which influence erosion of burned areas and also the recovery of the burned vegetation that it is unwise to generalize about either. The author's estimate of 1 500 cu yd per sq mile annually from areas unburned for 10 yr, or more, seems a generous maximum.

From experience in the Experimental Forest the writers submit some rates of erosion, in Table 16(*a*), based upon actual catchments in tight reservoirs

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[26] "Experiments with Critical Depth Flumes for Measurement of Flow in Débris-Laden Streams", by H. G. Wilm, J. S. Cotton, and H. C. Storey, California Forest and Range Experiment Station, presented at the Pacific Coast Meeting of the Am. Geophysical Union (Section of Hydrology), at the California Inst. of Technology, Pasadena, Calif., February 1, 1936. (Publication pending.)

TABLE 16.—EROSION RATES AND RUN-OFF IN CHAPARRAL WATER-SHEDS, 1934–1935

Water-shed	Area, in square miles	Elevation, in feet	(a) THE ENTIRE RAIN SEASON, 1934–35			(b) THE GREAT STORM, JANUARY 1, 1934		
			Precipitation, in inches of depth	Erosion rate, in cubic yards per square mile	Condition of water-shed	Precipitation, in inches of depth	Maximum run-off, in cubic feet per second per square mile	Erosion in cubic yards per square mile
Bell No. 3......	0.097	3 500	33	500*	Burned in 1919	11.8	34	62
Fern No. 1......	0.055	5 000	41	30	Not burned †.	12.6	26	65
Fern No. 2......	0.063	5 000	41	10	Not burned †.	12.7	22	19
Fern No. 3......	0.083	5 000	41	12	Not burned †.	12.6	19	24
Average....	.....	.....	......	....	....	12.4	25	42

* This high rate resulted from a small local landslip probably started by construction disturbances.

† Not burned for fifty years or more.

for the entire rain season of 1934–1935. From the great New Year's storm ending January 1, 1934, the rates measured in the same experimental water-sheds were as shown in Table 16(*b*). These extremely low run-off and erosion rates, when compared with the rates cited by Mr. Eaton for the same storm on the Mt. Lukens burned area, indicate that the quantitative influence of good vegetative cover is considerable. The burned water-sheds received an average of 12.5 in. of rain, yielded a maximum run-off of 580 cu ft per sec per sq mile (Pickens Canyon), and an erosion rate of approximately 67 000 cu yd per sq mile (Haines Canyon). Thus, the run-off ratio, as of covered to burned area, is 1:23, and the erosion ratio, 1:1 600.

Loss of soil by erosion in the burned area shown in Fig. 16 approached 100 000 cu yd per sq mile. The inter-shrub spaces proceeded to form gullies almost unhindered, because of the wide spacing between burned shrubs, their slow sprouting, and the sparseness of ground-cover vegetation between the shrub clumps. Ground-cover plants are lacking because the soil-stored seeds which might have escaped burning, were subsequently washed down the slope by sheet erosion.

The unburned water-sheds cited in Table 16(*a*) may be considered too small to be significant. Erosion measurements for large unburned water-sheds are not available, but run-off records from such water-sheds for the great

TABLE 17.—RUN-OFF FROM LARGE UNBURNED WATER-SHEDS, STORM OF JANUARY 1, 1934

Water-shed	Area, in square miles	Rainfall, total, in inches	Run-off, in cubic feet per second per square mile
Arroyo Seco....................................	16.4	11 to 17*	58
Eaton..	6.5	13.8	51
Big Santa Anita................................	10.5	13 to 19*	54
San Dimas......................................	18.3	10.8†	53

* Range of catchment in various rain-gauges.

† Catchment determined by statistical analysis of the catch in 150 rain-gauges distributed over the area.

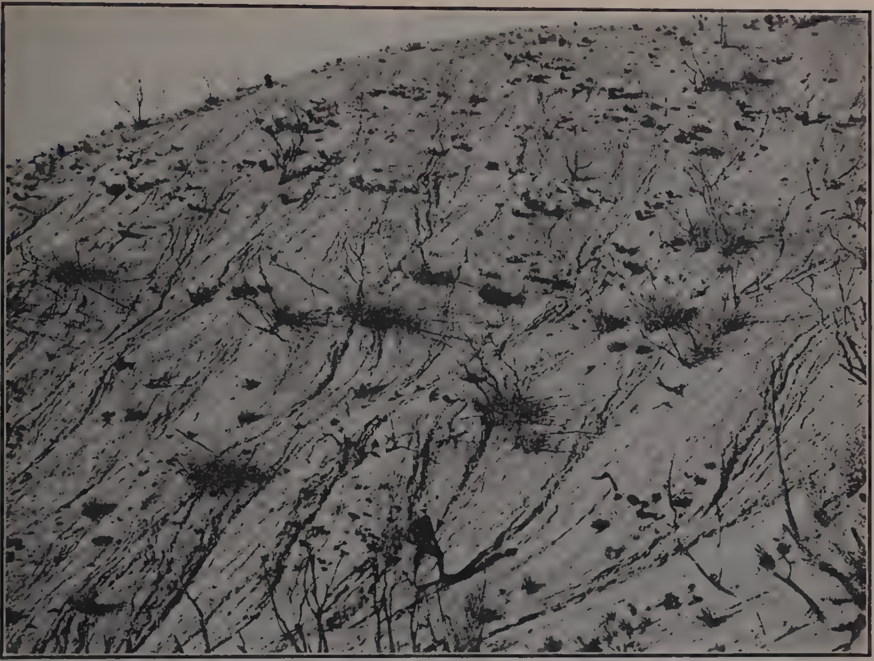

FIG. 16.—LOSS OF SOIL BY EROSION IN BURNED AREA (PHOTOGRAPHED SECOND WINTER AFTER FIRE) APPROACHED 100 000 CUBIC YARDS PER SQUARE MILE.

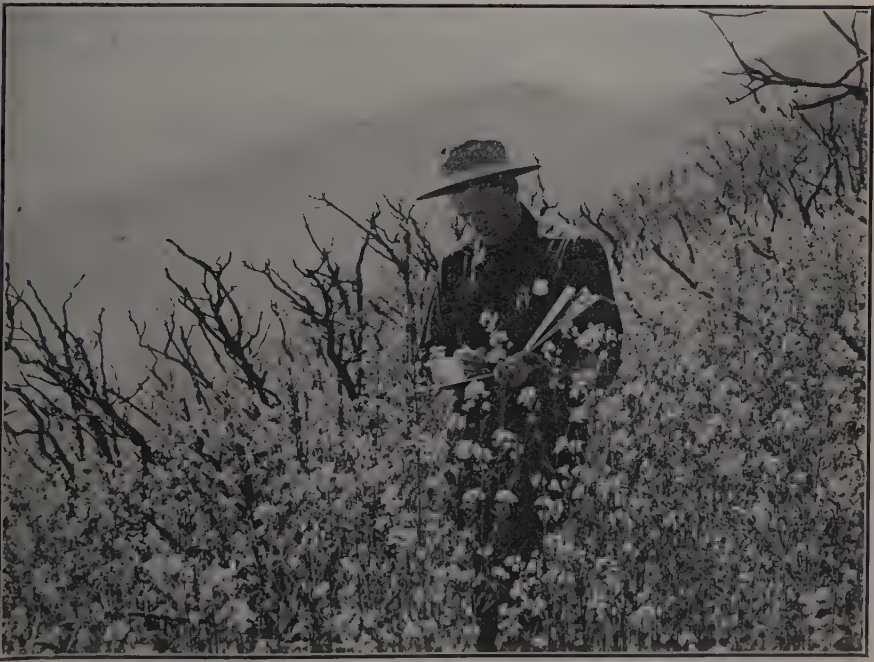

FIG. 17.—DENUDED AREAS SOWN TO MUSTARD AT RATE OF 5 POUNDS OF SEED PER ACRE.

storm clearly reflect a regulatory influence of vegetative cover similar to that on the small water-sheds (see Table 17). The run-off rates from these water-sheds, lying in the same mountain range as the burned area, receiving the same or heavier rainfall, and having similar geologic structure and topography, nevertheless yielded peak run-off only one-tenth (or less) as high as that from the burned area. The logical deduction that the controlled run-off from the forest-covered areas could have produced proportionately little erosion is substantiated by the fact that the water, observed during the storm, was almost clear of suspended matter.

As a means of reducing sheet erosion from burned water-sheds, the Forest Service has adopted from agriculture the idea of sowing a quick cover-crop and has found the lowly mustard to be the plant best suited to the purpose. The cost per acre, when sown by hand at the rate of 5 lb of seed per acre, averages $1.25. Inaccessible country is sown by airplane at $0.90 per acre, but the work is dangerous and the seed distribution is likely to be irregular. Quantitative data on the soil-holding effects are not yet available, but Fig. 17 shows a typical mustard stand, six months after sowing, in which the mustard with a density of 6 plants per sq ft, has filled the inter-shrub spaces with soil-binding roots, while the tops protect the soil from driving rain and wind. The oven-dry weight of mustard plants, returned as protective litter to the burned soils, ranges from 2 000 to 6 000 lb per acre the first year, and the seed production by this first crop is from 200 to 1 500 lb per acre. It is expected that the mustard will ultimately be crowded out by the recovery of the chaparral. At that stage there is a remote possibility that sufficient seed may remain stored in the soil for many years to produce a protective cover crop after some future fire.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THE RELATION OF ANALYSIS TO STRUCTURAL DESIGN

Discussion

BY MESSRS. I. K. SILVERMAN, BRUCE G. JOHNSTON, AND HAROLD E. WESSMAN

I. K. SILVERMAN,[16] JUN. AM. SOC. C. E. (by letter)[16a]—Character of stress rather than its magnitude is the subject of this paper. Since most of the papers concerning structural analysis appearing in the technical literature of the last ten years have dealt with the technique of stress determination, this paper deserves special attention.

From the author's point of view structural analysis is suggested as a measuring device for efficiency. Thus, if loads were fixed, a statically determinate structure which was built along the pressure line corresponding to the given loads would be as efficient as possible as far as structural action was concerned.

The fundamental assumption made in structural analysis is to the effect that under the action of loads the geometry of the structure remains unchanged as regards span, depth, angles, etc. Under this assumption both statically determinate and indeterminate structures are analyzed for their primary stresses. In the analysis of a statically determinate system it is unnecessary to know the make-up of the members or their physical properties, except to be sure that they are such that deformations caused by the loads do not have an appreciable effect on the dimensions of the structure. It would be difficult, for instance, to determine by ordinary methods the stresses in a structure made of rubber if the loads were such as to cause large deformations. For indeterminate structures with constant moduli of elasticity the make-up of the members must be known since "hybrid" action is present in practically all cases. The writer is acquainted with a truss, continuous over three spans, in which the make-up of the lower chord members adjoining the intermediate piers had a marked effect on the stresses in other members.

NOTE.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in October, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1935, by Messrs. L. J. Mensch, and Russell C. Brinker; and January, 1936, by Marshall G. Findley, Assoc. M. Am. Soc. C. E.

[16] Care, U. S. Bureau of Reclamation, Denver, Colo.

[16a] Received by the Secretary January 3, 1936.

The fixed arch, which is a very common structure, exhibits hybrid action (as defined by the author) to a high degree. Under the action of temperature the stresses at the springing are greatly effected by the thickness at the crown. Fig. 4 illustrates the case in which an increase in the depth of the crown of an arch did not necessarily lead to a reduction in unit stress.[17] This structural action is one which ordinarily cannot be predicted. In a circular arch it is found that temperature stresses can be reduced more effectively by increasing the central angle than by changing the strength of the member.

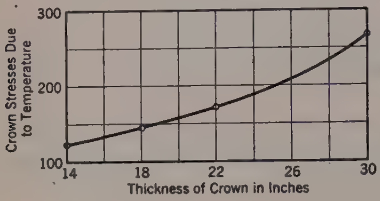

FIG. 4.

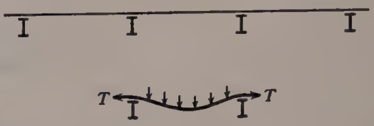

FIG. 5.

A type of action that may be classified under participation stresses, although the stresses produced are load-carrying stresses, is that which is illustrated by the modern suspension bridge in which advantage is taken of the increase in the sag of the cable in computing the maximum cable pull. Another case of the same type of action is shown in Fig. 5 for the continuous plate subjected to a normal load and supported as shown.

A study of the development of structural forms furnishes a fertile field of illustrations of the types of action classified by the author, and the history of the development of these forms illustrates how structural analysis has helped in the production of frames that have primarily normal structural action. A study of this development would be as valuable a part of the curriculum of engineering schools as are those parts which deal with the technique of stress determination.

BRUCE G. JOHNSTON,[18] JUN. AM. SOC. C. E. (by letter).[18a]—A logical and concise classification of the factors underlying the relation between structural analysis and design has been presented by the author. The paper deals principally with determinate and indeterminate frames, but the same logical thought applies as well to the study of any structural part, or detail, however small. The "Participation Stresses", as defined by the author, become somewhat analogous to localized or concentrated stresses in a structural detail, to which increasing attention has been drawn in recent years.

Every structural element is designed to perform a definite service without failing or deflecting too far under the working load that it carries. Obviously, the safe working load on the structure should be a specified fraction of the load at which the "limit of structural usefulness" is reached. The "limit of structural usefulness" is usually not a function of the maximum stresses alone, although it may be so in special cases. Many studies have been made with regard to theories of failure under combined stress at a point[19], with none as yet generally satisfactory; but the results of such a theory, even if available, would usually have only an indirect relation, if any, to the general failure of any particular type of structure as a whole.

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[17] "Design of Symmetrical Concrete Arches", by Charles S. Whitney, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 998.

[18] Instr., Dept. of Civil Eng., Columbia Univ., New York, N. Y.

[18a] Received by the Secretary January 11, 1936.

Studies of the stress distribution in a mathematically idealized elastic body present varied and interesting mathematical problems. Although solutions of many of these problems form the basis and historical background of structural analysis, the continued search for new solutions leads often into byways of small practical significance, and the subject of "stresses" may receive disproportionate attention at the expense of a more complete consideration of general structural behavior up to the point of failure.

The "limit of structural usefulness" of any structure may be determined experimentally by plotting the load-deflection diagram and determining, by one of the various arbitrary means available, the yield point of the structure as a unit. If every element of material in the structure is still in the elastic range as the "limit of structural usefulness" is approached—then an exact and complete mathematical analysis gives a close approach to the solution of the particular problem. However, in many types of structures, made wholly or in part of a ductile material such as structural steel, a slight "giving" of the material in local parts often effects a favorable redistribution of stresses. In such cases the local deformations may cause a relatively negligible permanent deflection in the structure as a whole. Although this fact is well known among structural engineers, its importance is sometimes lost sight of by those who may be interested only in elasticity or photo-elasticity. In some cases, however, as exemplified by both the first and second failures in the design and erection of the Quebec Bridge, the neglect of secondary factors caused failure. The conclusion must be that mathematical analysis and engineering judgment are inseparable factors, one without the other being of no great use.

The following outline gives examples of some of the principal zones of elastic or plastic action within which the limit of structural usefulness may fall:

A.—Limit of structural usefulness reached within, or at the limit of, the elastic range of the material:

(1)—Limited by elastic stresses;
(2)—Limited by elastic deflections; and
(3)—Limited by elastic buckling.

B.—Limit of structural usefulness not reached until after parts of the structure have passed the elastic limit:

(1)—Plastic yielding localized and not significant;
(2)—Plastic yielding localized at a critical point, leading to failure; and,
(3)—Limited by plastic buckling.

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[19] "Failure Theories of Materials Subjected to Combined Stresses", by Joseph Marin, Jun. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., August, 1935, p. 851.

Numerous examples of different structural types may be found to fall under each of the preceding headings.

Both the allowable working load and the limit of structural usefulness may be further conditioned by any of the following:

(1)—The hazard and type of use to which the structure is to be put;
(2)—The type of loading—static, dynamic, repeated, or alternating;
(3)—Physical properties and uniformity of quality of material; and
(4)—Precision of fit and type of fabrication.

Continued research is needed to further the correlation of these factors by studying the entire "load history" of all types and variations of structural elements which are used in current engineering practice. Further development of the theory of elasticity in combination with a consideration of plasticity will greatly increase the effective usefulness of many precise theoretical analyses.

HAROLD E. WESSMAN,[20] ASSOC. M. AM. SOC. C. E. (by letter).[20a]—Many readers accustomed to looking for strings of equations and tables of numerical data as earmarks of distinction will fail to realize the value of the structural classification proposed in this extremely important paper. It is well to keep in mind that the paper although qualitative and philosophical in nature, betrays extensive quantitative research of a high order. Such research, involving both analysis and design, must necessarily support the significant conclusions reached by the author.

The inclusion of definite numerical data might confuse or minimize the importance of the basic philosophy. Nevertheless, in view of the conclusions about analysis made by the Committee of the Structural Division, on Wind-Bracing in Steel Buildings[21], the writer hopes that the author will reinforce his remarks on rectangular wind frames by specific examples. In certain types of statically indeterminate structures, analysis and design cannot be considered apart from one another with any assurance of intelligent conclusions. Structures are generally built in accordance with design specifications that impose certain controlling conditions. Moreover, grades, clearances, construction methods, connecting details, and variations in the physical constants of the material, also establish limitations that must be considered in analysis if one wants a structure to act the way it is figured to act.

As the author states the concepts involved in his classification of types of structural action are not new. Nevertheless, the profession is greatly indebted to Professor Cross for proposing a logical classification that tends to clarify the relation of analysis to structural design. Regardless of a difference of opinion as to terminology or designation, one cannot help but feel that the author has given structural engineers a "shot in the arm" that will lift many of them out of the rut of conventional thought.

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[20] Assoc. Prof. of Structural Eng. and Mechanics, Coll. of Eng., Univ. of Iowa, Iowa City, Iowa.

[20a] Received by the Secretary January 27, 1936.

[21] See p. 397.

Undoubtedly, experienced designers have sensed the existence of "hybrid" structural action, but for the most part they have had neither the time nor the inclination to go into that phase of analytical research which brings out the full significance of interaction between various units. More often than not the practicing engineer is hurried in the preparation of plans for a definite structure. If computations show that certain members are overstressed, the tendency is to increase areas or moments of inertia in proportion to the over-stress, without taking time for a final check analysis. This is quite satisfactory if normal structural action emphatically predominates. Unfortunately, many indeterminate structures are of such a nature that a change in area or in moment of inertia of a member also means a change in the load or in the moment resisted by that member.

The writer recalls difficulties encountered in the design of some of the columns supporting the continuous plate-girder viaducts built by the South Park Commissioners over the Illinois Central Railroad tracks along the Lake Front, in Chicago, Ill. The maximum width of the columns in a direction parallel to the longitudinal axis of the bridge was limited by horizontal track clearances established by the Illinois Central Railroad Company. The minimum width was limited by the slenderness ratio for primary compression members in the governing specifications. The columns were rigidly connected to the continuous girders and were subject to dead and live load reaction, dead and live load moment, temperature moment, traction moment, and a transverse wind moment.

A preliminary design section based upon a preliminary analysis was found to be overstressed when all factors were combined. Conventional procedure indicated that more section was needed. The area was increased in proportion to the over-stress, but when a check analysis was made, the column was still overstressed; therefore, the effect of the various factors was then studied separately.

When area was added to the cross-section the unit stress from normal dead load plus live load was decreased; but the unit stress from the bending moment caused by dead load and live load remained practically the same, because increasing the area increased the stiffness, thereby drawing more moment to the column. Moreover, as long as the width of the column was kept constant, the unit stress from temperature moment remained constant regardless of any change in the area of the column. The effect of traction was very small. Decreasing the width of the column was considered. This decrease would have lowered the unit stress, but on the other hand it would also lower the allowable limiting stress because of an increase in slenderness ratio.

Evidently, curves showing the relation between unit stress and area, moment and moment of inertia, or unit stress and width of column, are important aids in selecting the final design. Other relationships may be dictated by the needs of the particular problem.

The author points out the existence of hybrid structural action, particularly in two-legged bents, rectangular wind frames, and reinforced concrete arches integral with their spandrel structures. The writer wishes to add the two-hinged suspension bridge of the usual type (not self-anchored) to the list. It may seem a far cry from a suspension bridge to a flitched beam; nevertheless, the two have much in common.

The cable and the stiffening truss participate in resisting total external bending moment at any section. If an unstiffened cable is used, the external bending moment at any section due to dead load and live load is equilibrated by $(H_w + H)\ (y + \eta')$, in which $H_w + H$ is the total horizontal component of cable stress and $y + \eta'$ is the total cable ordinate to the deflected cable at the section considered; η' represents the deflection of the unstiffened cable away from normal position. Since the cable itself is flexible, the cable shape must conform to the equilibrium polygon for the load acting on the span.

When a stiffening truss is added to the cable, the deflection, η', at any section is reduced to a value, η, depending upon the stiffness of the truss. If the truss has infinite stiffness, the cable will be forced back through the entire distance, η', to the normal position under dead load and mean temperature. The change in moment at the section is then $(H_w + H)\ \eta'$. To preserve equilibrium at the section, this expression will then be the moment in the stiffening truss—if the truss is infinitely stiff.

For a flexible truss, the cable will be forced back only a part of the distance, η', and the moment in the truss will be some proportion of $(H_w + H)\ \eta'$. If cable stretch and side-span interaction are ignored, the proportion is given by Equation (1):

$$M = \frac{M_t}{M_t + (H_w + H)\ \eta'}\ (H_w + H)\ \eta' \quad \ldots\ldots\ldots\ldots\ldots (1)$$

in which M_t is the moment that would be induced in the stiffening truss if it were bent to the deflection curve of the unstiffened cable. In general terms, M_t is given by Equation (2):

$$M_t = C_1\ E\ I\ \eta' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which C_1 is a constant depending upon loading conditions.

The moment given by Equation (1) does not include the moment in the truss caused by cable stretch from stress and temperature and change in center sag due to side-span interaction. This additional moment is expressed by Equation (3), also in general terms:

$$M = C_2\ \frac{E\ I}{l^2}\ df \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which C_2 is a constant depending upon the variation in suspender loading, and df is the change in center sag due to cable stretch from stress and temperature, and to side-span interaction.

From Equations (1) and (3), quite accurate values of the maximum moment in the stiffening truss at the quarter-point and at the center may be found for preliminary design purposes. The derivation and application of these equations are not given in this discussion.

Equation (1), particularly, emphasizes the existence of hybrid action. As the stiffness of the truss is increased, the moment taken by the truss increases. If greater stiffness is obtained by keeping the chord area constant and increasing the depth of the truss, the unit stress may actually increase. This seems paradoxical but computations bear it out. Note what happens to the maximum positive moment in the main span truss of the Manhattan Suspension Bridge when the moment of inertia of the truss is varied.

Fig. 6 shows this graphically. The abscissa labeled, *I*, corresponds to 43 900 in^2-ft^2, the equivalent moment of inertia of the main span truss as designed. The corresponding moment is 119 000 000 ft-lb. If the value of *I* is doubled, the maximum moment increases. On the other hand, if *I* is halved or quartered, the moment decreases rapidly. The relation is not linear, however.

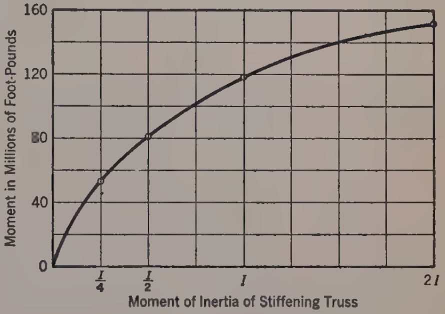

FIG. 6.—RELATION BETWEEN MAXIMUM POSITIVE MOMENT AND MOMENT OF INERTIA OF MAIN SPAN STIFFENING TRUSS OF MANHATTAN BRIDGE.

If the chord areas are kept constant and a variation in *I* is obtained by changing the depth of the truss, the curve in Fig. 6 may supply the basis for a new curve in which unit stress is plotted as an ordinate and depth of truss as an abscissa; or, depth may be kept constant and a variation in *I* obtained by varying the area of the cross-section. A curve of unit stress against chord area may then be plotted.

Fig. 6 emphasizes the fact that the most economical stiffening truss is no stiffening truss whatever. What shall be done about local grade changes and curvatures in the roadway, however? The question "stares one in the face"; so a stiffening truss is put on the cable. How stiff shall it be made? What is the limiting grade change or curvature? Who knows? Here is a field for extensive research.

Fig. 6 reveals that there is no one design for the stiffening truss of a suspension bridge. There are any number of possible designs. Conventional analysis fails almost completely as a guide to proper proportions. Studies such as these and the author's "Conclusions" reinforce a growing conviction on the part of the writer that one of the most fertile fields of research in structural engineering is the development of accurate and rapid preliminary design methods. It is most certainly a field that appeals to the practicing engineer.

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DISCUSSIONS

TUNNEL AND PENSTOCK TESTS AT CHELAN STATION, WASHINGTON

Discussion

BY S. P. WING, M. AM. SOC. C. E., AND J. N. BRADLEY AND H. P. EVANS, JR., JUNIORS, AM. SOC. C. E.

S. P. WING,[4] M. AM. SOC. C. E., AND J. N. BRADLEY,[4] AND H. P. EVANS,[4] JUNIORS, AM. SOC. C. E. (by letter).[4a]—Most hydraulic formulas are essentially empirical and can only be used with confidence within the range for which their coefficients have been determined experimentally. The author's tests of a 14-ft penstock, with velocities as high as 14 ft per sec, give very welcome information at the extreme limit of existing experimental data.

It is to be regretted that the author failed to give his field observations. This has tended to confuse observational with derived results which, combined with a somewhat unusual method of analysis, has made it difficult to compare these tests with the work of others. Since the results of this test, together with those from a very few others, form the basic data from which coefficients for larger installations must be extrapolated, it is thought that a few notes concerning the accuracy of the tests and a brief analysis of the results in a form more readily comparable with the work of others will be of value.

The experiments and analyses consist essentially in the determination of the hydraulic losses in a long pressure conduit with various fittings. Piezometric measurements were made on five sections as follows, lengths being given in pipe diameters: (1) Intake to vent, 18 diameters; (2) vent to Adit 1, 385 diameters; (3) Adit 1 to Adit 2, 274 diameters; (4) Adit 2 to surge tank, 98 diameters; (5) and surge tank to scroll-case.

Section (1) consists of the trash rack, 10.8 diameters of concrete-gathering tube, 4.1 diameters of 14-ft steel pipe to the center line of a horizontal butterfly valve, and 1.1 diameters of steel pipe from the valve to the vent. High-speed water-level recorders measured the hydraulic grade at the lake and the vent.

NOTE.—The paper by Ellery R. Fosdick, Esq., was published in October, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February 1936, by W. A. Hill, Esq.

[4] Engrs., U. S. Bureau of Reclamation, Denver, Colo.

[4a] Received by the Secretary January 27, 1936.

Section (2) consists of 1.7 diameters of steel pipe, and 383.3 diameters of 14-ft straight concrete-lined tunnel.

Section (3), from Adit 1 to Adit 2, consists of 274 diameters of 14-ft straight concrete-lined tunnel. The hydraulic grades at both adits were measured by mercury manometers connected to single taps in the conduit.

Section (4), from Adit 2 to the surge tank, consists of 98 diameters of 14-ft concrete-lined tunnel, followed by 28.5 diameters of 14-ft single-butt, strap-riveted, steel pipe which contains a 30° bend with a radius of 4.20 diameters. The surge-tank riser, 25 diameters down stream from the bend, is 14 ft in diameter and makes a vertical angle of approximately 19° with the penstock. Water elevations were read by a single piezometer connecting into the riser.

Section (5), from the surge tank to the scroll-case, consists, first, of 712 ft of steel pipe, varying in diameter from 14 ft at the surge tank to 12 ft 6 in. at the wye connection (average, 54 diameters of 13.25-ft pipe). Part of the pipe is single-butt and the remainder is double-butt, strap-riveted. At the symmetrical wye, which has a central angle of 45°, the pipe is reduced from 12 ft 6-in. to two 8 ft 10-in. branches. From the center line of the wye to the center line of a 29° bend with a radius of 7.5 diameters, is a distance of about 6.2 diameters, and from the bend to the center line of the balanced valve is 7.6 diameters, the total length from the wye to the center line of the valve being 13.8 diameters of 8 ft 10-in. pipe. A ring piezometer is connected to a dampened mercury manometer, 15 ft (1.7 diameters) from the center line of the valve. The piezometer connections are 18 in. up stream from the 6 ft 6-in. opening to the turbine scroll-case in the wall of the reducer which follows the valve.

Prior to the hydraulic tests reported in the paper, efficiency tests were made on the turbines using the Gibson method to determine the discharge. For this purpose a differential manometer was connected to a single tap at a point about 2 diameters down stream from the end of the wye and 1.0 diameter up stream from the bend, and to a single tap at a point about 4.5 diameters down stream from the end of the bend and 0.5 diameter ahead of the valve, the length of included pipe being 8.8 diameters. Simultaneously with the differential pressure readings, turbine head readings were made at the ring piezometer as well as such electrical measurements as were necessary to compute the generator efficiency. Each unit was tested with the other unit closed down, variations in load being obtained by controlling the wicket-gate openings of the unit being tested. From data thus obtained discharge curves were plotted for varying generator outputs, heads, and gate-openings. These curves were then used in the later tests to determine the mean velocities.

Since the author does not give the details of the efficiency tests, it is difficult to estimate their accuracy. As previously described the locations of the pressure taps were near fittings which caused abnormal pressure and velocity distributions in the flowing water, thus making high precision difficult.

The total energy passing a given cross-section of a conduit per second is expressed by: $w \int \left(z + \frac{p}{w} + \frac{v^2}{2\,g} \right) v \, da$. Where unusual conditions exist, such as near fittings, the law expressing the variation of each of these quantities throughout the cross-section is unknown; for example, the ring piezometer, from which the head on the turbine was obtained, is only about 7 diameters below a combined vertical and horizontal bend, 1 diameter below the balanced valve, and only 0.22 diameter up stream from the scroll-case. At this section. the stream lines would not be parallel to the pipe walls.[5] If actual velocity and pressure traverses were to be made across the section the following might represent the magnitude of the corrections required in the nominal values to obtain the true energy grade: $\pm$ 1.0 ft $+$ 0.1 h_v $+$ 0.2 h_v $=$ 0.3 h_v $\pm$ 1.0 ft.

For the maximum h_v measured (5.3 ft), this correction equals 0.6 ft to 2.6 ft, and is from 0.16% to 0.70% of the head on the turbine. Although this correction is nothing but an estimate, certain of the tests discussed later give confirmatory evidence. It should be noted that such a percentage correction if necessary also applies directly to the efficiency determination.

Considering the difficulties under which the discharge measurements were obtained originally and the errors involved later in transferring electrical measurements at various gate-openings to velocities, it is believed that for the hydraulic tests an error of at least 3% in the velocities can be expected.

The difficulties which have just been discussed in determining the true energy grade at a point where the stream lines are not parallel and where the velocity and pressure distribution throughout the flowing stream are unknown, apply to three other of the piezometric locations. The effect of this condition will be discussed in connection with the tests.

Section (1).—Intake to Vent.—This section contains trash rack, gathering tube, and friction losses plus an apparent drop in pressure across a butterfly valve. The vent at which the hydraulic grade was measured is only 0.5 diameter down stream from the leaf of the open gate. From Italian tests[6], in which the differential drops in hydraulic grade across butterfly valves were measured, the drop across the valve may be estimated as 0.45 h_v for the author's test, the velocity head being that for the 14-ft pipe. The friction losses in the intake section may also be estimated as 0.1 h_v. Subtraction of the sum of the friction loss and valve drop from the nominal energy loss indicates that the net trash rack and gathering tube entrance losses certainly do not exceed 0.05 h_v and are probably less. The losses given in Fig. 3, which approximate 0.60 h_v, are, therefore, not net intake losses, but consist largely in the drop across the butterfly valve.

Sections (2) and (3).—Vent to Adit 2.—The writers consider that the tests made on Sections (2) and (3), which include 659 diameters of a straight 14-ft concrete conduit, are by far the most accurate and valuable of the author's data and are deserving of more discussion than he has given them. There are few tests of large pipe at high velocities, as free from errors due to the effects of bends and fittings as these tests. Two methods of treating the data are possible: Either Section (3) alone can be analyzed, the difference between the readings of Adit 1 and Adit 2 giving the energy loss directly, as at these sections the velocity distributions are the same; or Section (2) can be combined with Section (3), and the loss over a much greater length of pipe can be used. In this latter case a correction must be applied to the drop in the hydraulic grade as measured to allow for the influence of the butterfly valve. Either of the foregoing two methods yields substantially the same results. However, since errors in piezometer readings affect the accuracy inversely proportional to the length of section in which the friction loss is determined, Sections (2) and (3) combined appear to give the more reliable results, even when the additional error involved in estimating the valve correction is considered. The method of making this estimate is as follows:

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[5] "Strömung in Spiral Gehausen", by Kranz, V. D. I., *Forschungs*, Heft 370.

[6] *Energia Electtrica*, November, 1933, p. 922.

If to the reading at the vent ($1.0\ h_v + 0.45\ h_v$) is added, the energy grade up stream from the valve is obtained. The energy loss due to the valve, which, of course, is less than the piezometer drop across it, occurs in Section (2) and may be estimated as $0.25\ h_v$. At Adit 2 the velocity profile is fully developed, so that the true energy head may be approximated by adding $1.05\ h_v$ to the piezometer reading. The net energy head loss in Sections (2) and (3) thus equals the difference in readings between the vent and Adit 2, plus $0.15\ h_v$. This valve correction may only be within $\pm\ 0.15\ h_v$ of the correct value, but since the measured loss approximates $7.0\ h_v$, the percentage error would only be of the order of 2 per cent. The probable error of the mean of sixteen field determinations of the friction drop is computed to be about 1 per cent. Combining all errors it is believed that the determinations of the friction factor have a probable error of not more than 4 per cent.

The best method for estimating and comparing hydraulic losses in pipes, which the writers have found, consists in plotting experimental results on logarithmic paper using friction factors, f, equal to $\frac{2\,g\,D\,H}{L\,V^2}$, as ordinates and Reynolds' numbers, $\frac{V\,D}{\nu}$, as abscissas. Good discussions of the theoretical basis for using these non-dimensional parameters have appeared elsewhere.[7] A very practical reason for using such plots, however, is that they provide an excellent means of assembling and comparing visually the entire range of experimental data. From them the errors probable in extrapolation, or in using formulas, may be easily estimated.

Fig. 13 is the plot of the experimentally measured losses between the vent and Adit 2 of the Chelan Tunnel, both the total measured loss and the friction factor, f, being plotted against the Reynolds' number. For comparison with this test the results of equally reliable experiments made on two of the tunnels of the Catskill Aqueduct and on the 18-ft tunnel of the Niagara Power Company, as given by Kemler, have been added. These data, obtained from straight concrete conduits, for which the concrete was poured against steel forms, approximating a mortar finish the surface of which was free from form marks, show a consistent trend of losses over a considerable range of

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[7] "Modern Conceptions of the Mechanics of Fluid Turbulence", by Hunter Rouse, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1936, p. 21; also, "A Study of the Data on the Flow of Fluids in Pipes", by E. Kemler, *Transactions* A. S. M. E., Hyd. 55-2.

Reynolds' numbers. They become of great value, therefore, for the purpose of extrapolating to Reynolds' numbers of from 30 000 000 to 80 000 000, a range which covers flow in pipe from 20 to 50 ft in diameter.

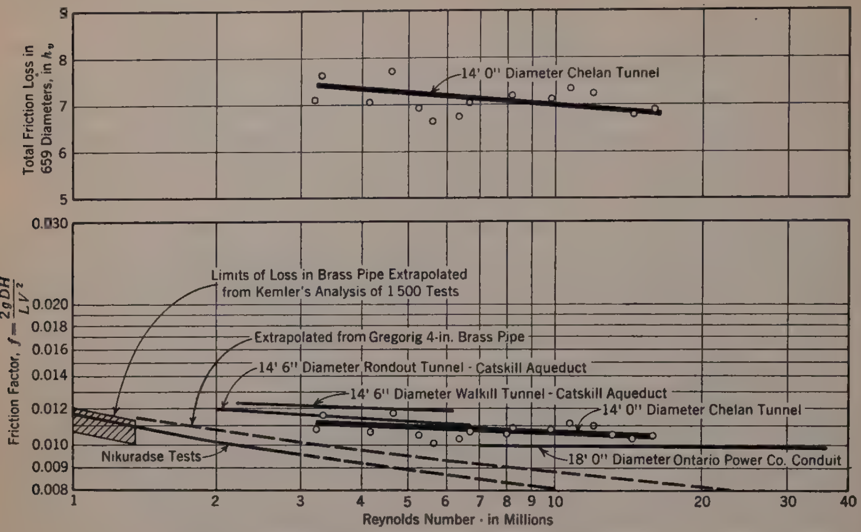

FIG. 13.—FRICTION LOSS IN CHELAN TUNNEL.

To aid in extrapolation, tests on smooth brass pipe at high Reynolds' numbers have also been plotted. As concrete pipe becomes larger and larger, its surface relatively tends to approach in smoothness that of the smaller brass pipe. The lower limit for losses in large pipe, therefore, may be expected to approach the losses found for brass pipe at similar Reynolds' numbers.

In his analysis of the losses in the concrete pipe, the author apparently used the measured loss between the vent and the surge tank and, from this, subtracted the estimated loss in the steel pipe, making no other correction. Table 3 indicates the differences in the two methods of treatment.

TABLE 3.—COMPARISON OF LOSSES IN PIPES

Velocity, in feet per second	Reynolds' number	KUTTER'S n		HAZEN-WILLIAMS' C	
		Author	Present analysis	Author	Present analysis
14	15 250 000	0.0112	0.0116	134	124
4	4 360 000	0.0128	0.0120	131	133

Although the author's values are within 2 to 9% of those given by the writers, the differences are large enough to be of significance in connection with the design of large pipe lines.

Section (4).—Adit 2 to Surge Tank.—This section contains losses due to friction in both concrete and steel pipe, a 30° bend, and an apparent pressure change due to expansion into the surge-tank riser. At Adit 2 the energy grade may be obtained by adding 1.05 h_v to the piezometer reading. At the surge-tank riser neither the average pressure nor the velocity head of the flowing water is known, therefore, the energy grade at this point cannot be computed directly. As an estimate the author assumed that, due to the acute angle between the riser and the penstock, the water level in the riser would be lowered about 0.1 h_v. Therefore, he obtained the nominal energy grade by adding 1.1 h_v to the surge-tank reading. The correctness of this assumption may be checked roughly by estimating the friction and bend losses in the steel pipe, adding the friction losses, as obtained from the experiment of the concrete pipe, and subtracting the sum from the energy grade at Adit 2. The following is the computation using the data from the author's Test No. 4:

At Adit 2:

Elevation of hydraulic grade, in feet.................... = 1 069.73
Mean velocity, 12.16 ft per sec; 1.05 h_v, in feet........... = 2.42

Energy grade, in feet................................ = 1 072.15

Loss Between Adit 2 and Surge-Tank Riser:

Experimentally determined friction loss in concrete pipe, 98D × 0.0106 h_v................ = 1.04 h_v
Estimated friction loss in steel pipe, 28.5D × 0.0147 h_v.............................. = 0.42 h_v
Estimated friction loss in bend............... = 0.06 h_v

Total energy loss, in feet................ = 1.52 h_v = 3.50

At Surge Tank:

Energy grade at Adit 2 less loss, in feet................... = 1 068.65
Velocity head, in feet........................... 1.05 h_v = 2.42

Computed hydraulic grade, in feet...................... = 1 066.23
Measured hydraulic grade, in feet...................... = 1 067.04

Re-gain of head, in feet....................... 0.34 h_v = 0.81

The average re-gain of velocity head at the surge tank, as determined by similar computations for all tests having a velocity of 1 ft, or more, was 0.15 h_v. Instead of a re-gain, the author assumed there would be a drop due to suction. The result is that in all his computations of the losses in Section (5) he used a surge-tank energy grade 0.2 h_v above that which the writers consider applicable.

Section (5).—Surge Tank to Scroll-Case Piezometer Ring.—This section contains the following losses: (1) Sudden contraction loss from surge-tank riser; (2) friction loss in 54 diameters of 13.25-ft. steel pipe; (3) wye loss;

(4) friction loss in 13.8 diameters of 8.83-ft. pipe; (5) 30° bend loss; and (6) balanced valve drop. Since the velocity and pressure distributions at the surge tank and the piezometer ring are both different and unknown, correction factors must be applied to the hydraulic grade drop in order to obtain the total true energy loss in this section. According to previous estimates the true energy grade at the surge tank is about 0.2 h_v less than, and that at the scroll-case, 0.3 h_v higher than, the grades assumed by the author, the combined corrections reducing the total loss as given by the author by about 25 per cent. These figures are applicable to the condition of equal discharge in each of the penstock branches.

Turning next to the author's methods of distributing the total energy loss from the surge tank to the scroll-case into its six component parts, and from these data deriving laws for the losses in a wye, in a bend, and in a steel pipe, the writers consider that in spite of the ingenious methods which were used to make the most of the meager information available, the conclusions must be considered to apply only to the particular installation tested with, moreover, the reservation that piezometric drops are involved and not true energy losses. Even to obtain experimentally the true losses for this single installation, six additional measurements would have been required. For example, to evaluate the losses due to the bends in the wye branch the author made use of the differential pressures measured in the turbine efficiency tests. As already indicated, in this case the absolute magnitude of the pressure drop obtained depended both on the particular location of the piezometers on the pipe sections and on the conditions of flow both up stream and down stream from the piezometer locations. The pressure drop actually measured approximated 0.15 h_v for the higher range of velocities tested. This is three times the energy loss estimated from various bend experiments which have been made on small pipes[8]. Although the results from the latter cannot be extrapolated with any confidence to a bend of the size tested by the author, on the other hand, results obtained under such unfavorable field conditions as existed in these tests can only be regarded with extreme caution.

It is believed the author's treatment of "eddy losses in the wye branch above the tie-plate" and of the losses through the balanced valve is inadequate. Perhaps a piezometric drop up stream from the tie-plate may be expected, but the major part of any energy loss must occur down stream from any obstruction. The author estimated the energy loss through the valve on the basis of the computed friction loss. An additional loss would be expected due to the curvature of the stream lines. On the whole, the writers consider that the best use which can be made of the author's field measurements in Section (5) is for the purpose of comparing the measured nominal total loss in energy due to a wye, 30° bend, and valve with that of a usual estimate.

On Fig. 14 the nominal total lost energy head from the surge tank to the piezometer ring, as obtained from Table 3, in terms of the velocity head existing in each of the 8.83-ft branches, has been plotted for various values of the ratio of the discharges in the two wye branches. Subtracting from these total losses, the friction loss in the steel penstocks, as estimated by the writers, two curves are obtained which give the sum of the contraction loss at the surge-tank riser, the wye loss, the 30° bend loss, and the balanced valve loss.

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[8] *Transactions*, Munich Hydr. Inst., *Bulletin 3*.

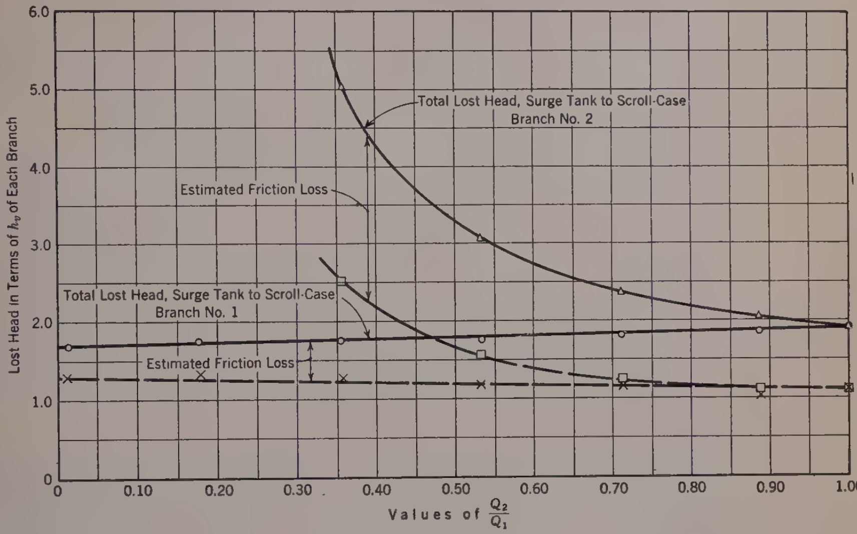

FIG. 14.—LOST HEAD FROM SURGE TANK TO SCROLL-CASE FOR FLOW OF 17 TO 1 125 CUBIC FEET PER SECOND, IN BRANCH NO. 2, AND FLOW IN BRANCH NO. 1 UNIFORM AT 1 125 CUBIC FEET PER SECOND.

It will be noted, from these curves, that for increasing values of $\frac{Q_2}{Q_1}$, the losses in Branch 2 decrease on account of the great reduction in the deceleration losses through the wye. When $\frac{Q_2}{Q_1} = 1$, the nominal loss is $1.1 \frac{V^2}{2g}$. The following is the writers' distribution of the energy losses for this case, expressed in terms of the velocity head of the branch:

Entrance loss at surge-tank riser................	$0.05\ h_v$
Wye loss	$0.20\ h_v$
Bend loss	$0.05\ h_v$
Apparent loss in valve........................	$0.25\ h_v$
Total	$0.55\ h_v$
Errors in estimates of losses and differences between true and nominal energy gradients at ends of section..............................	$0.55\ h_v$
Total	$1.10\ h_v$

In conclusion, the writers feel that although the author's experiments did not provide adequate data from which true energy losses in bends, wyes, or steel pipe could be determined, they did give required operating data, and information on the friction losses in large concrete pipe which long will be of great value to other engineers.

The courtesy of the author in supplying the writers with information additional to that in the original paper has been much appreciated. To him and to the Washington Water Power Company are due the thanks of all hydraulic engineers for the expenditure of the time and money required in making and analyzing field tests on a large conduit and presenting the results to the profession.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

TAPERED STRUCTURAL MEMBERS: AN ANALYTICAL TREATMENT

Discussion

BY MESSRS. E. G. PAULET, J. CHARLES RATHBUN, AND HALVARD W. BIRKELAND

E. G. PAULET,[16] JUN. AM. SOC. C. E. (by letter).[16a]—The formula for the variation of moment of inertia introduced by Professor Max Ritter and presented by Walter Ruppel, Assoc. M. Am. Soc. C. E.[17], is directly reducible to Equation (1) suggested by the authors. When the shape exponent is made equal to 1, a special case of Professor Ritter's formula results.[18] The writer has used the formula for this special case in the analysis of tapered members encountered in structures, such as swing and bascule girder bridges, and rigid-frame bridges.

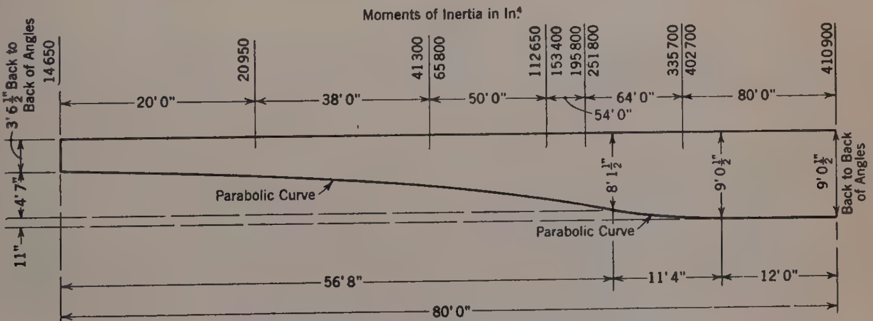

FIG. 14.—CHARACTERISTICS OF ONE ARM OF A SWING SPAN.

NOTE.—The paper by Walter H. Weiskopf and John W. Pickworth, Assoc. Members, Am. Soc. C. E., was published in October, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1936, by Messrs Fred. L. Plummer, and LeRoy W. Clark.

[16] Bridge Design Engr., State Highway Comm., Baton Rouge, La.

[16a] Received by the Secretary January 6, 1936.

[17] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 159; also, "Analysis of Continuous Frames", by E. B. Russell, Second Edition, p. 23; San Francisco, Calif., 1934.

[18] *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio.

In Fig. 14 is shown one arm of an actual girder swing span with equal arms, the bottom chord of which is on a tangent and two reverse curves, the ratio of the curves being one-fifth, although one-fourth is also employed. Using the authors' notation and letting Fig. 15 represent the tapered member with its elastic behavior similar to that of the swing span arm, the ordinates of the influence line for the center reaction are given by the following equations:

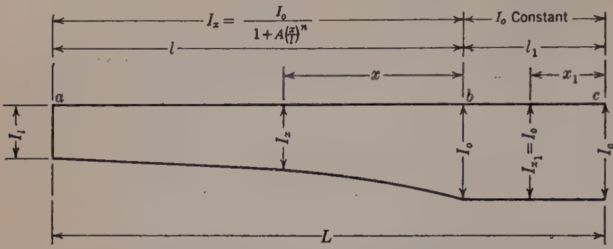

FIG. 15.—TAPERED MEMBER USED FOR ANALYSIS OF SWING SPAN.

$$\begin{aligned} y_{(x=0\ to\ l)} = {} & \frac{3}{\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \Bigg\{ \frac{L^2\,l}{2} + l^3 \Bigg[-\frac{1}{6} \\ & + A\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\Bigg] + \frac{x\,(l^2 - L^2)}{2} + x^2 \Bigg[-\frac{l}{2} + \frac{x}{6} \\ & - \frac{A\,x^n}{l^{n-1}}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) - \frac{A\,x^{n+l}}{l^n}\left(-\frac{1}{n+2} + \frac{1}{n+3}\right)\Bigg]\Bigg\} \quad ..(146) \end{aligned}$$

and,

$$y_{(x1=0\ to\ l1)} = 1.0 - \frac{x_1^2\,(3\,L - x_1)}{2\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \quad ..(147)$$

From Equations (146) and (147), the positive center reaction due to a uniform load, $w = 1.0$ per unit length, covering both arms is:

$$\begin{aligned} R_{\text{center}} = {} & \frac{6\,l^2}{\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \Bigg\{ \frac{L^2}{4} + l^2 \Bigg[-\frac{1}{24} \\ & + A\,l^2 \Bigg(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \\ & - \frac{\left(\frac{1}{n+1} - \frac{1}{n+2}\right)}{n+3} - \frac{\left(-\frac{1}{n+2} + \frac{1}{n+3}\right)}{n+4} \Bigg)\Bigg]\Bigg\} \\ & + \left[2 - \frac{l_1^2\,(4\,L - l_1)}{4\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]}\right] l_1 \quad .....(148) \end{aligned}$$

and the negative end reaction due to a uniform load, $w = 1.0$ per unit length, covering one arm is:

$$R_{\text{end}} = -\frac{\text{Equation (148)} - L}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (149)$$

in which L is the length of one arm.

Equation (1) of the paper was used in the derivation of Equations (146) to (149)[19]. For a shape exponent, $n = 1.0$, the following design values were adopted: $I_0 = 403\,000$ in.^{4}; $I_l = 14\,650$ in.^{4}; $l = 60$ ft; $l_1 = 20$ ft; and $A = 26.5$. Fig. 16 shows the paths of the actual I-curve and substitute I-curve. A value

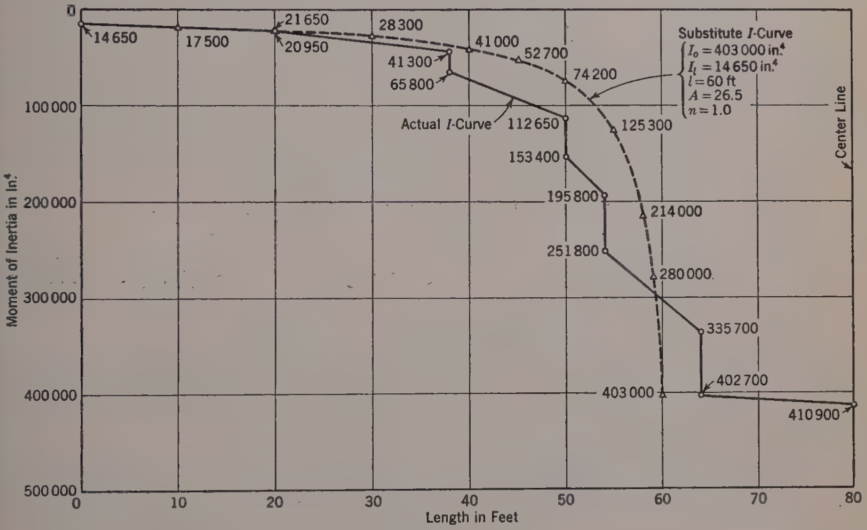

FIG. 16.

of $l = 64$ ft for the length of the tapered member would be preferable, provided a suitable shape exponent, n, were determined. However, since Equations (146) to (149) had first been derived with a value of $n = 1.0$, and the substitute I-curve with such a value would rise quickly as shown in Fig. 16, a length, $l = 60$ ft, for the tapered member was chosen as a rational value.

After substitution of the design values in Equations (148) and (149), the center reaction is found to be $1.471\,w\,L$, and the negative end reaction, $-\,0.1178\,w\,L$. By the moment-area method, the center reaction is $1.472\,w\,L$, and the negative end reaction is $-\,0.1179\,w\,L$. Therefore, when a shape exponent, $n = 1.0$, is used, and judgment is exercised in the selection of a rational value for l, similar cases may be thusly approached and results obtained satisfactory for all practical purposes. However, since the shape exponent, n, appears in Equations (146) to (149), its rational value could well be substituted in these equations. The most probable value of n, obtained by the least square method, is 1.64, and the arithmetic mean value is 1.55 ($l = 64$ ft being used). Obviously, such values for n can be obtained only when the sections at various points along the tapered member are already known. Consequently, a value of n that is suitable for the majority of cases would be desirable. The authors mention a value of 2 for n. If $n = 2$, then $l = 64$ ft, $l_1 = 16$ ft, $L = 80$ ft, and $A = 26.5$, are substituted in Equations (148) and (149), the center reaction is found to be 1.451 $w\,L$ and the negative end reaction is $-\,0.1129\ w\ L$. These values of reactions compare very favorably with those found by the moment-area method, and a shape exponent, $n = 2$, would be a satisfactory value for this class of structures.

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[19] The complete discussion, with supporting derivation, has been filed for reference in Engineering Societies Library, New York, N. Y.

A comparison of the values for the center reaction and the negative end reaction obtained by considering I to be variable, with those obtained when I is considered constant, shows that the center and end reactions of the former case are, respectively, 18% and 88% greater than in the latter case. For uplift, the consideration of the tapered member analysis is important. This analysis yields final design moments which are greater than those obtained by the analysis when I is considered constant, the excess varying from 9% at the quarter-point from the end to 4% at the center support.

The deflection at each end of the tapered arms of a swing span with equal arms due to a concentrated load, P, applied at each end is:

$$y = \frac{P}{E I_0}\left[\frac{L^3}{3} + A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right] \ldots\ldots(150)$$

Let $P = 15\,000$ lb; $E = 30\,000\,000$ lb per sq in.; $I_0 = 403\,000$ in.^{4}; $L = 80$ ft; $l = 64$ ft; and $A = 26.5$. Substituting these values in Equation (150), the end deflection, y, is found to be 1.03 in. when $n = 1.64$, 1.09 in. when $n = 1.55$, and 1.62 in. when $n = 1.0$. By the moment-area method, $y = 1.11$ in.

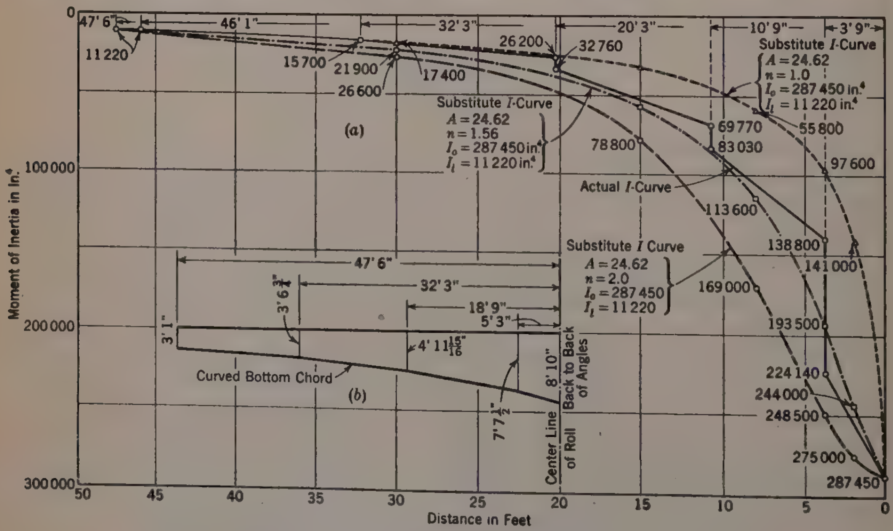

FIG. 17.—ONE-LEAF, 95-FOOT, ROLLING LIFT BASCULE SPAN.

From a practical viewpoint, the deflection obtained from Equation (150) is found more quickly than that obtained by the moment-area method. If rational values for the shape exponent, n, are desired, a family of curves similar to Fig. 2 may be prepared and used in conjunction with a diagram of the actual *I*-curve of the member drawn to the same scale as the *n*-curves. By properly placing the *n*-curves over the actual *I*-curve, as shown in Fig. 1, a satisfactory value for the shape exponent, *n*, may be chosen.

In Fig. 17 are given the general dimensions and moments of inertia at various sections of one leaf of a double-leaf, rolling, lift, bascule girder bridge. The taper modulus, *A*, is 24.62. The most probable value of the shape exponent, *n*, is 1.56, and the arithmetic mean value is 1.42. The graph of the three substitute *I*-curves having shape exponents, 1.0, 1.56, and 2.0, shows the relative departure of these curves from the actual *I*-curve.

The shear, *V*, carried by the center lock of a double-leaf bascule bridge, the main girders of which are tapered and loaded with a concentrated load, *P*, distant *k l* from the center line of trunnion or roll (the leaves being assumed cantilevered from the trunnion or roll, and $I_x = \dfrac{I_0}{1 + A\left(\dfrac{x}{l}\right)^n}$) is:

$$V = P\left[\frac{k^2}{2} - \frac{k^3}{6} + A\,k^{n+3}\left(-\frac{1}{n+2} + \frac{1}{n+3}\right) + A\,k^{n+2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right] \div 2\left[\frac{1}{3} + A\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right] \text{........(151)}$$

and adopting a shape exponent of $n = 2.0$,

$$V = P\left[\frac{2\,(15\,k^2 - 5\,k^3) + 5\,A\,k^4 - 3\,A\,k^5}{40 + 4\,A}\right] \text{...........(152)}$$

For leaves having a uniform moment of inertia (that is, with $A = 0$),

$$V = \frac{P}{4}\,(3\,k^2 - k^3) \text{ ....(153)}$$

From Equation (152), the limit of *V* as *A* approaches infinity is:

$$\frac{P}{4}\,(5\,k^4 - 3\,k^5) \text{........................(154)}$$

In Fig. 18 are plotted the influence lines for *V* when $A = 0$ and $A \to \infty$, also when $A = 25$ for the leaf shown in Fig. 17. The marked departure of

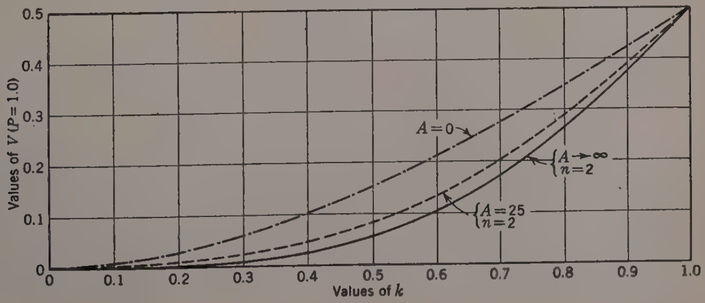

FIG. 18.—INFLUENCE LINES FOR *V*.

the curves for $A = 0$ and $A \to \infty$ near the center of the leaf where it is a maximum at $k = 0.624$, indicates whether the leaf should be analyzed as a member of constant moment of inertia or as a tapered member. The choice is a matter of individual preference. Considering the uncertainty of end-fixity of the leaves at the trunnion, or the center line of the roll, and the value of the positive taper modulus, A, as being in general above 20, the writer has adopted the following equation for the influence line for V,

$$V = \frac{P k^2}{8} (3 - k + 5 k^2 - 3 k^3) \quad \text{.............} (155)$$

The influence line given by Equation (155) is for $A = 10$, and plots midway between the curves for $A = 0$ and $A \to \infty$.

The shear, V, carried by the center lock, due to a uniform load, w, per unit length, extending throughout the length, l, of one leaf is: For a tapered member:

$$V = \frac{w\,l \left[\frac{1}{4} + A\left(\frac{1}{n+1} - \frac{3}{n+2} + \frac{3}{n+3} - \frac{1}{n+4}\right)\right]}{4\left[\frac{1}{3} + A\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \quad \text{...} (156)$$

and for a member of uniform moment of inertia,

$$V = \frac{3\,w\,l}{16} \quad \text{.......................} (157)$$

From Equation (157), when $n = 2$ and $A = 10$,

$$V = \frac{5\,w\,l}{32} \quad \text{.......................} (158)$$

Furthermore, when $n = 2$ and $A \to \infty$, the limit of,

$$V = \frac{w\,l}{8} \quad \text{.......................} (159)$$

It may be seen from Equations (157), (158), and (159), that as the value of A increases, the value of V rapidly converges toward $\frac{w\,l}{8}$. For a concentrated load, P, on one leaf, a rapid convergence also exists.

With the aid of Equation (1), of the paper, an analysis of various types of structures having tapered members will show directly the relative behavior of their component parts. The derivation of formulas for sought values, while not difficult, may be lengthy and tedious at times; however, for the simpler types of structures which are of frequent application, an analytical treatment should certainly receive consideration. The authors[20] have already contributed much information in this direction.

In his analysis of symmetrical parabolic arches, F. Guerrini[21] uses a function for the substitute I-curve similar to the authors' Equation (4), and which is:

$$I_x \cos \theta_x = \frac{I_0}{1 - 0.5\left(\frac{x}{l}\right)^2} \quad \text{.................(160)}$$

in which θ_x is the angle that the tangent to the parabolic axis of the arch makes with the horizontal at any point distant x from the crown, and l is the half-span length, I_0 and I_x being, respectively, the moments of inertia of the arch at the crown and at any point distant x from the crown.

Professor J. Rieger[22] has analyzed members of variable moment of inertia involving straight haunches. His analysis is exact but usually leads into complicated expressions unless certain approximations are adopted. He has also considered such approximations and has compared the results with those obtained by exact analysis. Tapered members with straight haunches may be analyzed with Equation (1) when a suitable shape exponent, n, is used. For positive taper modulus, $A = 4$ to $A = 10$, a value of 2 for n would be satisfactory for all practical purposes. For straight haunches, with positive taper modulus, $A = 0$ to $A = 20$, the curve of arithmetic mean values for n is approximately given by, $n = \sqrt{\frac{A + 0.14}{3.5}} + 0.8$, and the curve of most probable values for n is approximately given by, $n = 0.63\sqrt{\frac{A + 0.14}{3.5}} + 0.874$. The variable error occurring when these approximate equations are used does not exceed 6 per cent.

The method of substitute I-curves is convenient for the direct analysis of structures having tapered members. When more than three statically indeterminate quantities are sought and, therefore, a direct solution becomes involved if not impracticable, recourse to a method such as the slope-deflection method or the Cross method of distributing fixed-end moments, is convenient. With the authors' method of substitute I-curves all the necessary coefficients entering into the slope-deflection and Cross methods may be computed and put in tabular[23] or graphical[18] form.

Although Equation (1) is Professor Ritter's general formula re-stated in a different form, its convenient presentation, its interpretation, and its applications, undoubtedly demonstrate the convenient adaptability and far-reaching possibilities of the method of substitute I-curves.

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[20] "Symmetrical Rigid Frames", and "Unsymmetrical Rigid Frames", by W. H. Weiskopf and J. W. Pickworth, Am. Inst. of Steel Construction, N. Y., 1934.

[21] "Calcul des Ponts avec Arc Parabolique et Tablier Supérieur", by F. Guerrini, *Le Constructeur de Ciment Armé*, Paris, July, 1935, p. 137.

[22] "Calcul des Constructions Hyperstatiques", by J. Rieger, Vol. II, 1931, p. 446, Paris, France.

[23] "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, 1932, p. 289.

J. CHARLES RATHBUN,[24] M. AM. SOC. C. E. (by letter).[24a]—As a study of one variation of the beam theory, this paper is of undoubted value. The authors have developed, rather completely, the theory as applied to a beam with moments of inertia that vary according to a given mathematical formula that contains three arbitrary constants. As the paper is well written, it should help very much in clarifying the beam theory.

From an engineering standpoint the writer feels that the value of this work depends upon its superiority over other methods both as to ease of application and of understanding. Whether the method can stand this test is open to question. Using this paper as a guide the analysis of a few beams would involve considerable study on the part of the engineer. Some labor could be saved in the authors' methods if one were to develop a series of tables or diagrams covering the various changes in the formula for taper as the assumed constants change. This would only be economical if the engineer were required to deal with a very large number of tapered beams to which this formula applies. The writer feels that such an addition should not be made as the merit of the paper lies in its theoretical rather than its practical interest.

The most obvious variation of the method used by the authors is to assume that the $\frac{1}{I}$-curve varies as a parabola, or that,

$$\frac{1}{I} = p_2 x^2 + p_1 x + p_0 \qquad (161)$$

in which p_2, p_1, and p_0 are constants so chosen that the curve will pass through predetermined points. By assuming more terms, the curve of Equation (161) may be made to pass through four or more points. Loads are usually concentrated, uniform, or, in a few cases, uniformly varying. These loads lead to moment curves with equations of the form:

$$M = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \qquad (162)$$

in which, again, a_3, a_2, a_1, and a_0 are constants. It follows at once that the $\frac{M}{I}$-curve is of this same form. Equation (162) is a form that is readily integrable as the only integral form used is $\int x^n \, dx = \frac{x^{n+1}}{n+1}$. This statement is true also of the expression in Equations (12), (13), (14), and (15), of the paper, and the expressions used in the Cross method. These parabolic curves can be made to fit the $\frac{M}{I}$-curve more closely than those of the authors as the irregularities of the curves (as in Fig. 1) need not be ironed out. If Equation (161) is substituted in Equation (52), the carry-over factor, C_{AB}, is obtained.

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[24] Associate Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

[24a] Received by the Secretary January 7, 1936.

As examples of the method of using Equations (161) and (162), consider the five cases cited by the authors, E being considered equal to unity.

Case 1.—Assume Equation (161) as the formula of the $\frac{1}{I}$ - curve, and defining i_A as $\frac{1}{I}$ at the point, A. When $x = 0$, $y = \frac{1}{I_A} = i_A$; when $x = l$, $y = \frac{1}{I_B} = i_B$; and, when $x = \frac{l}{2}$, $y = \frac{1}{I_C} = i_C$, the origin being taken at the left end. Consequently, $i_A = p_0$; $i_B = p_2 l^2 + p_1 l + p_0$; and, $i_C = \frac{p_2 l^2}{4} + p_1 \frac{l}{2} + p_0$. From these equations it follows that, $p_0 = i_A$; $p_1 = \frac{i_C - i_B - 3\, i_A}{l}$; and $p_2 = \frac{2\, i_B - i_C - 2\, i_A}{l^2}$.

The $\frac{M}{I}$ - curve becomes, for a uniform load: $\frac{M}{I} = \left(\frac{w\, l\, x - w\, x^2}{2}\right) (p_2\, x^2 + p_1\, x + p_0)$; and, by double integration:

$$y = \frac{w\, l}{2}\left(p_0 \frac{x^3}{6} + p_1 \frac{x^4}{12} + p_2 \frac{x^5}{20}\right) - \frac{w}{2}\left(p_0 \frac{x^4}{12} + p_1 \frac{x^5}{20} + p_2 \frac{x^6}{30}\right) + C\, x \quad \text{..(163)}$$

The constant, C, is obtained by placing $y = 0$ and $x = l$.

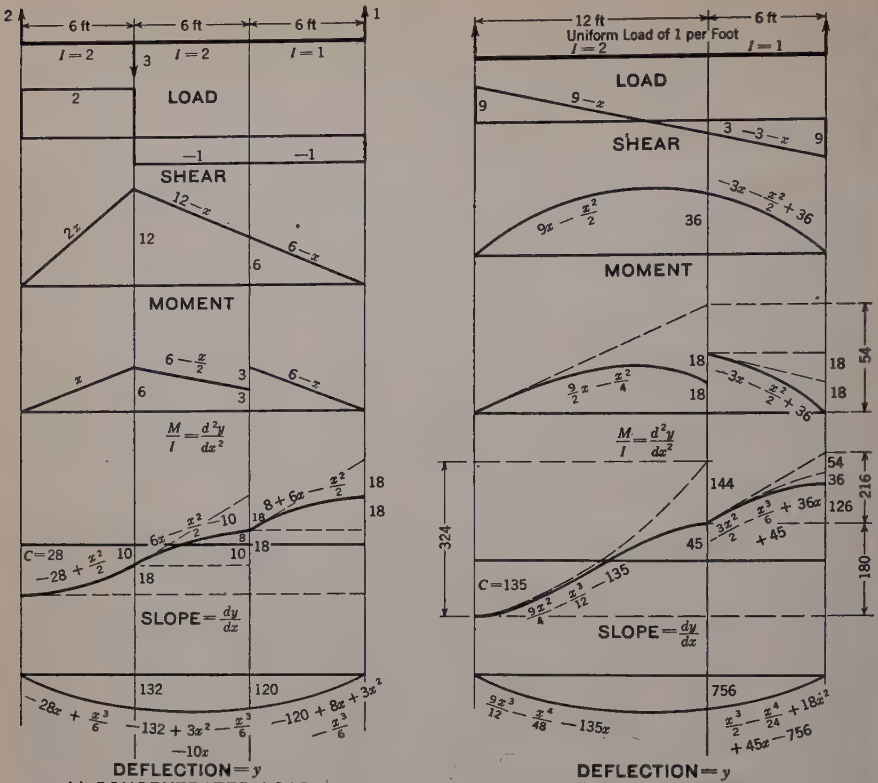
Case 2.—In this example, i_A may equal i_B as suggested by the authors, where $p_0 = i_A$; $p_1 = \frac{i_C - 4\, i_A}{l}$; and $p_2 = \frac{4\, i_A - i_C}{l^2} = \frac{-p_1}{l}$. On the other hand, one may shift the origin to the center taking advantage of the symmetry of the equations. Then the $\frac{M}{I}$ - curve becomes $\left(\frac{w\, l^2}{8} - \frac{w\, x^2}{2}\right) (i_C + p x^2)$; and, by double integration:

$$y = \frac{w\, l^2}{16}\, i_C\, x^2 + \frac{w(i_A - 2\, i_C)}{24}\, x^4 - \frac{w\,(i_A - i_C)}{15\, l^2}\, x^6 + C\, x \quad \text{.....(164)}$$

and the constant can be obtained by letting $x = \frac{l}{2}$ and $y = 0$.

Case 3.—This is the same as Case 2, or Case 1, depending upon the shape of the curve—that is, depending on whether or not it has zero slope at Point C.

Case 4.—In this case the authors make the approximation of substituting a continuous curve for a discontinuous one, as shown in Fig. 1. The I-curve is not continuous. In Fig. 19, the writer has developed numerical examples for both a concentrated and a uniform load in order to show that the conventional method presents no difficulties; nor does it involve excessive labor. For example, referring to Fig. 19(a), $18\, C = \frac{18 \times 6}{3} + 18 \times 6 + \frac{36 \times 6}{2} - \frac{18 \times 6}{3} + 36 \times 6 + \frac{36 \times 6}{2} - \frac{18 \times 6}{3}$; and $C = 28$. Furthermore, at the third-points, the ordinates to the deflection curve are: $\frac{6^3}{6} - 28 \times 6 = -132$; and $-132 + 3 \times 6^2 - \frac{6^3}{6} - 10 \times 6 = -120$.



(*a*) CONCENTRATED LOAD (*b*) UNIFORMLY DISTRIBUTED LOAD

FIG. 19.—ALTERNATE SOLUTION, CASE 4.

Referring to Fig. 19(*b*), $18\ C = \frac{324 \times 12}{3} - \frac{144 \times 12}{4} + 180 \times 6 + \frac{216 \times 6}{2} - \frac{54 \times 6}{3} - \frac{36 \times 6}{4} = 2\,430$. $C = 135$. Furthermore, at the third-point indicated, the ordinate to the deflection curve is: $\frac{9 \times 12^3}{12} - \frac{3 \times 12^4}{144} - 135 \times 12 = -756$.

This method has the advantage that it is theoretically exact. All the computations are given here or in the illustration. With more complicated numbers the use of the slide-rule will reduce the work to approximately that of the examples.

The equations are written on the curves for those who prefer the multiple-integration method and the areas are divided for those who prefer to integrate by computing areas. With the curves the origin is taken at the left of the section for which the curve is written. When this system is followed the constant of integration is obtained by placing x equal to the section length in the section to the left of the one being integrated. For the constant of the slope curve in the extreme left section, C, the area under the entire curve considering the temporary axes as at the beginning of the left end of the curve, is computed. This area is divided by the length of the beam.

Case 5.—This problem is only the simple beam theory as given in texts on the subject.

It is the writer's opinion that the substitution of a parabolic curve for the reciprocal inertia curve is simpler to understand, is as readily applied, and is more elastic than the method given by the authors. By using a parabola of a power higher than the second, the curve can be made to check at more than three points. By changing the equation where sudden changes occur in the inertia curve, these changes can be considered mathematically.

The method given herein does not involve any ideas that are not familiar to those who are engaged in work on the beam theory. If the writer has demonstrated that it is superior to that given by the authors he has not detracted greatly from a very excellent mathematical treatise on the comparatively complicated subject of tapered beams.

HALVARD W. BIRKELAND,[25] JUN. AM. SOC. C. E. (by letter).[25a]—The authors have written a commendable paper on a subject that has increased in importance during the last ten to fifteen years. Two references are given for equations similar to those presented in the paper; to these may be added the names of A. Strassner[26], Walter Ruppel[27], Assoc. M. Am. Soc. C. E., and E. B. Russell[28]. There are also several other references for tapered structural members in general.[29]

In their "Conclusion", the authors state: "It is hoped that the method of substitute I-curves will thus aid in the development of classes of structures which have been hampered in the past by mathematical difficulties." It is quite true, and pathetic, that some engineers have difficulties even in the most elementary mathematics. However, the writer believes, from his experiences and from those of others, that the use of haunched members has been retarded not so much by mathematical difficulties as by a lack of geometrical interpretations of the beam constants. It is fortunate that many engineering terms and problems may be reduced to their elements, and there is no advantage in ignoring the physical significance of those elements. This is not only the writer's observation. The same opinion was expressed editorially[30] in 1935, although it was written in connection with a rather general article on structural analysis:

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[25] Junior Engr., U. S. Bureau of Reclamation, Denver, Colo.

[25a] Received by the Secretary January 14, 1936.

[26] "Neuere Methoden", by A. Strassner, Band I, Third Edition, Berlin, 1925.

[27] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points", by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E.; Discussion by Walter Ruppel, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

[28] "Analysis of Continuous Frames by the Method of Restraining Stiffnesses", by E. B. Russell, Second Edition, San Francisco, Calif., 1934.

[29] "Continuous Frames of Reinforced Concrete", by Hardy Cross, and N. D. Morgan, Members, Am. Soc. C. E., N. Y., 1932; "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, Fourth Edition, N. Y., 1932; "Concrete, Plain and Reinforced", by F. W. Taylor, S. E. Thompson, and E. Smulski, Members, Am. Soc. C. E., Vol. 2, Fourth Edition, N. Y., 1928; and, "The Modified Slope Deflection Equations", by L. T. Evans, Jun. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 28, 1932.

[30] *Engineering News-Record*, October 17, 1935, p. 551.

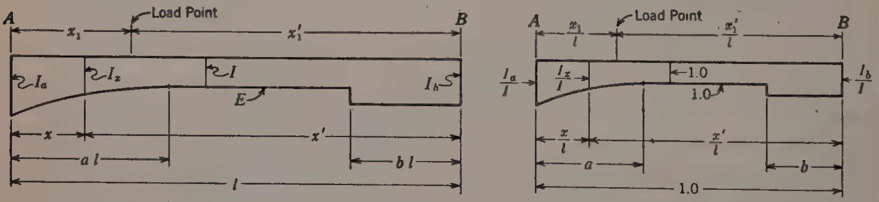
> "Just as every complex operation is made up of simple elements, so all phases of engineering consist of elementary facts and relations. Much of the engineer's skill lies in his grasp of these elements and his ability to recognize them in the particular problem with which he may be dealing. Nowhere is this truer than in the field of structures. Here the engineer's success at all times depends on how intimately he understands elementary physical facts and behavior."

With the preceding in mind, the physical meanings of the F-term will be presented. In this discussion the following fundamental F-terms will be considered: F_1, F_2, F_3, F_4, and F_5. An inspection of Equations (36), (37), (40), and (41) shows that $F_6 \equiv F_4$ and $F_7 \equiv F_5$. so far as any general discussion is concerned, and all other F-terms may be obtained by the proper integrations of F_4 and F_5. To illustrate the last statement, Equation (88) may be obtained from Equation (86) thus:

$$F_8 = \int_{k=0}^{k=1.0} F_4 \, dk \ldots\ldots\ldots\ldots\ldots\ldots\ldots (165)$$

Before giving the geometrical interpretations, it is necessary to introduce a fictitious beam, the "unit beam", so called for reasons which will be apparent later.

Definitions of the Actual Beam and the Unit Beam.—An actual beam is a beam or a member as it is in a structure, with all its actual dimensions, regardless of end restraints and loads. An example of an actual beam is shown in Fig. 20(a). Each actual beam has a unit beam which corresponds to it.



(a) ACTUAL BEAM (b) UNIT BEAM

FIG. 20.

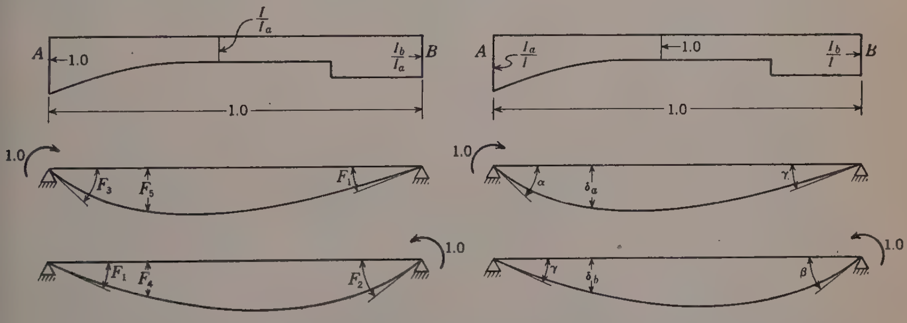
All dimensions of this unit beam are proportional to those of the actual beam: Longitudinal dimensions are reduced by the factor, l, usually the span length; the modulus of elasticity at every point is reduced by the factor, E, usually the modulus of elasticity of the material; and the moment of inertia at every point is reduced by the factor, I, usually the moment of inertia at the minimum section of the beam. An example of a unit beam is shown in Fig. 20(b). The beam in Fig. 20 is chosen as an illustration because it embodies dissymmetry, and constant section as well as gradual and abrupt changes in section, and will signify a beam with any arbitrary variation in moment of inertia.

The unit beam was defined by deriving it from the actual beam. Imagine the procedure reversed, and it is seen that one unit beam may be the source of any number of actual beams. Another unit beam may be the source of another group of actual beams. Unit beams, therefore, may be said to be a criterion of actual beams.

Credit for originating the unit beam concept should go to Mr. Ruppel, who states:[31] "When values for special loadings or beams are determined, it is suggested that they be computed for, or reduced to, coefficients for a unit load on the span of unity with I and E equal to unity."

Geometrical Interpretations of F-Terms.—Consider a simple-span unit beam in which the reduction factors are the span length, the modulus of elasticity of the material, and the moment of inertia at End A, as shown in Fig. 21(a). Then F_3 = the rotation at A due to a unit moment at A; F_2 = the rotation at B due to a unit moment at B; F_1 = the rotation at B due to a unit moment at A, or the rotation at A due to a unit moment at B; F_5 = the rotation at A due to a unit load at the load point, or the deflection at the load point due to a unit moment at A; and F_4 = the rotation at B due to a unit load at the load point, or the deflection at the load point due to a unit moment at B. The double definitions of F_1, F_5, and F_4 are due to Maxwell's law of reciprocal displacements.

The foregoing study of the physical meanings of the F-terms suggests that better symbols might have been chosen. The writer, therefore, proposes the following substitutions: α for F_3; β for F_2; γ for F_1; δ_a for F_5; and δ_b for F_4. The writer also proposes that the reduction factor for moment of inertia be I_c, the minimum moment of inertia of the member. This latter suggestion will make many of the authors' equations more symmetrical, and will simplify the relations between their paper and most of the material that has been published before. The relations between the authors' and the writer's symbols are illustrated in Fig. 21. The mathematical relations are:



(a) BY THE AUTHOR (b) BY THE WRITER

FIG. 21.—SIMPLE-SPAN UNIT BEAMS, WITH DEFLECTION CURVES AND BEAM CONSTANT.

$$\frac{F_3}{\alpha} = \frac{F_2}{\beta} = \frac{F_1}{\gamma} = \frac{F_5}{\delta_a} = \frac{F_4}{\delta_b} = \frac{I_A}{I_c} \ldots\ldots\ldots\ldots\ldots (166)$$

There are several books, booklets, and articles which have tabulations of beam constants for variously haunched beams, and these constants are usually

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[31] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 161.

TABLE 2.—CONVERSION FORMULAS FOR BEAM CONSTANTS (ARRANGED VERTICALLY).

(See Footnotes 2, 26, 27, 28, and 29 for complete reference.)

Authors	F_3	F_2	F_1	F_5	F_4
Writer's simple span unit beam..	$\frac{IA}{I}a$	$\frac{IA}{I}\beta$	$\frac{IA}{I}\gamma$	$\frac{IA}{I}\delta_a$	$\frac{IA}{I}\delta_b$
Strassner.........	$\frac{J_a}{J}\frac{a_a}{3}$	$\frac{J_a}{J}\frac{ab}{3}$	$\frac{J_a}{J}\frac{\beta}{6}$	$\frac{J_a}{J}\frac{\beta S_a}{6}$	$\frac{J_a}{J}\frac{\beta S_b}{6}$
Ruppel...........	$\frac{I'}{I}(1-u)p$	$\frac{I'}{I}(1-v)q$	$\frac{I'}{I}up=\frac{I'}{I}vq$	$\frac{I'}{I}ps$	$\frac{I'}{I}qt$
Russell...........	$\frac{I_f'}{I}\frac{1}{4K_f(1-C_nC_f)}$	$\frac{I_f'}{I}\frac{1}{4K_n(1-C_nC_f)}$	$\frac{I_f'}{I}\frac{C_f}{4K_f(1-C_nC_f)}=\frac{I_f'}{I}\frac{C_n}{4K_n(1-C_nC_f)}$	$\frac{I_f'}{I}\frac{F_f+C_fF_n}{4K_f(1-C_nC_f)}$	$\frac{I_f'}{I}\frac{F_n+C_nF_f}{4K_n(1-C_nC_f)}$
Cross and Morgan.	$\frac{IA}{I}\frac{1}{S_a(1-r_ar_b)}$	$\frac{IA}{I}\frac{1}{S_b(1-r_ar_b)}$	$\frac{IA}{I}\frac{r_b}{S_a(1-r_ar_b)}=\frac{IA}{I}\frac{r_a}{S_b(1-r_ar_b)}$	$\frac{IA}{I}\frac{M_a+r_bM_b}{S_a(1-r_ar_b)}$	$\frac{IA}{I}\frac{M_b+r_aM_a}{S_b(1-r_ar_b)}$
Turneaure and Maurer*	$\frac{I_1}{I_2}\frac{B}{AB^2+AC^2}$	$\frac{I_1}{I_2}\frac{B}{AB^2+AC^2}$	$\frac{I_1}{I_2}\frac{C}{AB^2+AC^2}$	$\frac{I_1}{I_2}\frac{BCD_1+C^2D_2}{AB^2+AC^2}$	$\frac{I_1}{I_2}\frac{BCD_2+C^2D_1}{AB^2+AC^2}$
Taylor, Thompson and Smulski*...	$\frac{I_1}{I}\frac{a}{3}$	$\frac{I_1}{I}\frac{a}{3}$	$\frac{I_1}{I}\frac{\beta}{6}$	$\frac{I_1}{I}\frac{\beta C_{v2}}{6}$	$\frac{I_1}{I}\frac{\beta C_{v1}}{6}$
Evans...........	$\frac{IA}{I_c}\frac{C_3}{C_1C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_1}{C_1C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_2}{C_1C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_3CAB+C_2CBA}{C_1C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_1CBA+C_2CAB}{C_1C_3-C_2^2}$
Large and Morris†	$\frac{1}{4S_a(1-r_ar_b)}$	$\frac{1}{4S_b(1-r_ar_b)}$	$\frac{r_b}{4S_a(1-r_ar_b)}=\frac{r_a}{4S_b(1-r_ar_b)}$	(No influence lines for fixed-end moments.)	

* Only for symmetrical beams.

† Symbols similar to those of Cross and Morgan have been assigned by the writer.

directly applicable only to the particular method with which they are presented. Because one set of tables may cover loads or haunches not included in other tables, it may be useful to have conversion formulas so that any one set of tables may be used for all methods. The writer has used such formulas since 1933 and has made them more complete from time to time. An extract is presented in Table 2, with the F-terms of the authors and the simple-span unit beam of the writer included. The table is arranged so that all terms in one column are equal. If the equations are solved for the constants used by the many other writers on the subject, the complete set of conversion formulas may be obtained.

Concerning the substitute I-curves in general, the writer advances the suggestion that there is little need for such substitutions, but what is needed is more accurate and more complete tables (not curves) of beam constants, similar to those presented by Strassner, Ruppel, and Russell.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

TRUSS DEFLECTIONS: THE PANEL DEFECTION METHOD

Discussion

By Messrs. A. A. Eremin, T. P. Noe, Jr., David A. Molitor, and Glenn L. Enke

A. A. Eremin,[9] Assoc. M. Am. Soc. C. E. (by letter).[9a]—Equations for computing truss panel deflections have been developed by Mr. Shoemaker in this paper. His method is more direct than the least work method with unit loading at each panel joint. However, the panel deflections may also be computed in one operation by means of elastic weights.[10] For example, at some chord joint of a truss, such as Point N, Fig. 13, the elastic weight is expressed by:

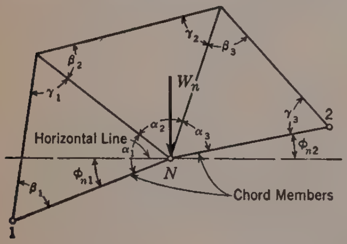

Fig. 13.—Elastic Weight of Truss Chord Joint.

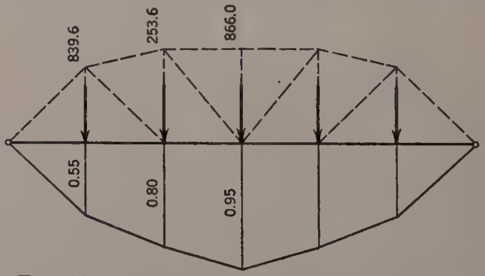

Fig. 14.—Deflection of Truss Lower Chord.

$$W_n = \delta\,\alpha_n + \frac{s_{n1}}{E}\tan\phi_{n1} + \frac{s_{n2}}{E}\tan\phi_{n2} \ldots\ldots\ldots\ldots (11)$$

in which, $\delta\,\alpha_n$ = the deformation of a chord angle at Joint N; ϕ_{n1}, ϕ_{n2} = angles between the chord and a horizontal line to the left and to the right of a

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Note.—The paper by Louis H. Shoemaker, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1936, by Messrs. David B. Hall, and E. Mirabelli; and February, 1936, by Messrs. William Bertwell, and Robert H. Hurlbutt.

[9] Associate Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[9a] Received by the Secretary December 30, 1935.

[10] Hütte: Des Ingenieurs Tashchenbuch, Bd. III.

joint; s_{n1}, s_{n2} = unit stresses in the chord members to the left and to the right of Joint N; and E = modulus of elasticity of the chord in tension and compression.

If the chord is horizontal the angles, $\phi_{n1} = \phi_{n2} = 0$. Therefore, the elastic weight is equal to:

$$W_n = \delta\,\alpha_n \qquad (12)$$

The angular deformation of the chord at Point N is computed from deformations of the angles formed by members converging at that point. The deformation of an angle such as α_2 in Fig. 13, is expressed as,[11]

$$E\,\delta\,\alpha_2 = (s_3 - s_2)\cot\beta_2 + (s_3 - s_1)\cot\gamma_2 \qquad (13)$$

in which β_2 and γ_2 are angles in the triangular panel opposite Joint N.

The elastic weights are considered as vertical forces at the chord joints. The force polygon of the elastic weights constructed through the end joints of a truss represents a deflection polygon of chord panels. The force polygon may be determined either graphically, or analytically, as the bending moment diagram of a beam sustaining the elastic weights at the spaces corresponding to the horizontal projections of the joints (see Fig. 14).

In Example 1 of the paper the elastic weights at the lower chord computed with Equation (12) are: $W_2 = 839.6$; $W_4 = 253.6$; and $W_6 = 866.0$.

Deflections at the lower chord panels determined as ordinates of the bending moment diagram in terms of $\frac{12}{30\,000}$ are: $\Delta_2 = 0.55$; $\Delta_4 = 0.80$; and, $\Delta_6 = 0.95$. These deflections are closely in agreement with those computed by the author (see Table 1). The elastic weights and deflections were computed with a standard 10-in. slide-rule. Various simplifications in the elastic weight method have been developed by Professor Müller-Breslau.[10,12].

The advantage of the elastic weight method over that introduced by Mr. Shoemaker is in the simplicity of the conventional rules for determining signs, which are the same as those for the stresses in the members of a truss. In the elastic weight method it is necessary to remember only two formulas (Equations (11) and (13)). Furthermore, the deformations of panel angles, computed in the elastic weight method may also be used for computing the secondary stresses in the truss.

T. P. Noe,[13] Jr., Jun. Am. Soc. C. E. (by letter).[13a]—A clearer conception of the panel deflection method as described by Mr. Shoemaker is gained by developing the basic equations by means of the Williot diagram. C. E. Ellis, M. Am. Soc. C. E., has pointed out in two papers[14] that the geometrical relationships established by a Williot diagram may be expressed in the form of algebraic equations which will yield an algebraic solution for those values previously obtained graphically from a Williot diagram drawn only for known changes in length of the truss members. The writer has had several occasions to make use of such an algebraic solution of the Williot diagram, notably in the determination of secondary stresses, but heretofore has not developed any standard expressions for typical truss panels. However, there is much merit in doing so. For unusual cases, however, the writer believes that the Williot diagram offers the best solution.

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[11] "Modern Framed Structures", by J. B. Johnson, C. W. Bryan, and F. E. Turneaure. 1917, p. 427.

[12] "Graphische Statik", Bd. II.

[13] Asst. Prof. of Civ. Eng., Univ. of North Carolina, Chapel Hill, N. C.

[13a] Received by the Secretary January 21, 1936.

[14] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 580; also *Engineering News-Record*, Vol. 112, April 26, 1934, p. 534.

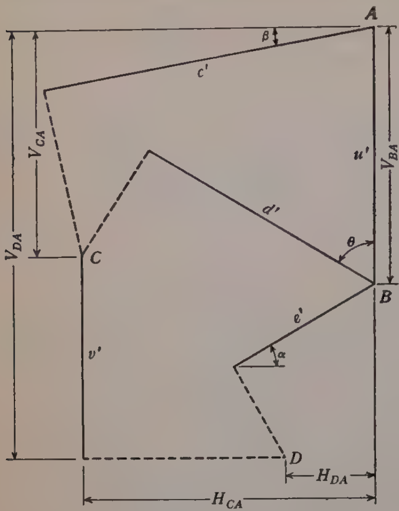

FIG. 15.

In Fig. 15 is shown the Williot diagram drawn for the typical panel considered by the author. This diagram is constructed on the basis of the assumptions made in the paper, namely, that Point *A* is fixed in position, Member *A B* is fixed in direction, and the change in length of each member is considered as a lengthening. From the geometry of Fig. 15 the following relations may be developed, Angles α, β, and θ referring to the slopes of the inclined members as indicated:

The vertical deflection of Point *C* relative to Point *A* equals,

$$V_{CA} = \frac{c' \sin \theta + u' \cos \beta \cos \theta - d' \cos \beta}{\sin \beta \sin \theta + \cos \beta \cos \theta} \quad \ldots\ldots\ldots\ldots (14)$$

the horizontal deflection of Point *C* relative to Point *A* equals,

$$H_{CA} = \frac{c' \cos \theta - u' \sin \beta \cos \theta + d' \sin \beta}{\sin \beta \sin \theta + \cos \beta \cos \theta} \quad \ldots\ldots\ldots\ldots (15)$$

the vertical deflection of Point *D* relative to Point *A* equals,

$$V_{DA} = V_{CA} + v' \quad \ldots\ldots\ldots\ldots (16)$$

the horizontal deflection of Point *D* relative to Point *A* equals,

$$H_{DA} = (e' \sec \alpha - v' \tan \alpha) - \frac{(c' \sin \theta - u' \sin \theta \sin \beta - d' \cos \beta) \tan \alpha}{\sin \beta \sin \theta + \cos \beta \cos \theta} \quad (17)$$

and the rotation of Member *CD* relative to Member *AB* equals,

$$R_{CD} = \frac{H_{CA} - H_{DA}}{\overline{C\,D}} = \frac{H_{CA} - H_{DA}}{v} \quad \ldots\ldots\ldots\ldots (18)$$

The sign convention and notation for change in length of a member are those of the paper.

Expressing the functions of the angles, α, β, and θ, in terms of the lengths of the members or their projections, and substituting these values in Equations (14), (15) and (17):

$$V_{CA} = \frac{c' \, c + u' \, w - d' \, d}{u} \qquad \text{(19)}$$

$$H_{CA} = \frac{(c' \, c - d' \, d + u' \, w) \, w}{u \, p} + \frac{(d' \, d - u' \, w)}{p} = \left(\frac{w}{p}\right)(V_{CA}) + \frac{(d' \, d - u' \, w)}{p} \qquad \text{(20)}$$

and,

$$H_{DA} = \frac{(e' \, e - v' \, v + v' \, w)}{p} + \frac{(c' \, c - d' \, d + u' \, w) \, w}{u \, p} - \frac{u' \, w}{p} - \frac{(c' \, c - u' \, u + u' \, w - d' \, d) \, v}{u \, p} \qquad \text{(21)}$$

Equations (19) and (20) are identical with Equations (1) and (2) of the paper.

Substituting the values of H_{CA} and H_{DA} given by Equations (20) and (21) into Equation (18):

$$R_{CD} = \left(\frac{1}{p}\right)\left(V_{CA} + \frac{d' \, d - e' \, e - v' \, w}{v} + v' - u'\right) \qquad \text{(22)}$$

Equation (22) is identical with Equation (3) of the paper.

It is evident that each of the panel points, C, D, and B, will undergo an additional deflection, both horizontally and vertically, if the reference member, AB, rotates through some small angle denoted by R_{AB}. These rotational displacements are similar in nature to those given by a Mohr diagram. It is easily seen that for the case considered these additional deflections are given by the following expressions:

$$V'_{CA} = (c \, R_{AB}) \cos \beta = p \, R_{AB} \qquad \text{(23)}$$

$$H'_{CA} = (c \, R_{AB}) \sin \beta = (u - w) \, R_{AB} \qquad \text{(24)}$$

$$V'_{DA} = (\overline{A\,D} \, R_{AB}) \left(\frac{p}{\overline{A\,D}}\right) = p \, R_{AB} \qquad \text{(25)}$$

and,

$$H'_{DA} = (\overline{AD} \, R_{AB}) \, \frac{(u - w + v)}{\overline{A\,D}} = (u - w + v) \, R_{AB} \qquad \text{(26)}$$

In order to illustrate the application of the panel deflection method to the case of an unsymmetrically loaded truss, the writer has computed the vertical deflections of a truss having the same dimensions and cross-sectional areas as that of the author's Example 1, but with an unsymmetrical loading. The values of the stresses in the members due to the assumed loading are recorded in Table 10. Panel points on the right of Member 5–6 are denoted by the corresponding panel point numbers appearing on the left with the prime attached.

TABLE 10.—VERTICAL DEFLECTIONS OF UNSYMMETRICALLY LOADED PRATT TRUSS

Deflection point (1)	Member (2)	Total stress, in kips* (3)	Change in length, inches $\times \frac{30\,000}{12}$ (4)	Relative vertical deflection given by Equation (19) (5)	Relative rotation of reference member given by Equation (22) (6)	Rotation relative to Member 5–6 = R (7)	Deflection due to rotation = p × Column (7) (8)	Total deflection with Member 5–6 fixed (9)	Transposed vertical deflections (10)	Deflections due to rotation of entire truss about Point 0 (11)	True vertical deflection Column (10) × Column (11) (12)	Vertical deflections, in inches = Column (12) $\times \frac{12}{30\,000}$ (13)
0	0–2	+653	+230.5	—980.2	....	....	....	—2 309.2	0	0	0	0
	0–1	—939	—358.5									
2	1–2	+350	+258.3	....	—14.34	—26.58	—797.4	—531.6	+1 777.6	+28.6	1 806.2	0.722
	1–4	+259	+192.6									
	2–4	+653	+230.5									
1	1–3	—845	—257.0	—278.4	....	....	....	—789.9	+1 519.3	+28.6	1 547.9	0.619
4	3–4	+164	+196.8	....	—12.24	—12.24	—367.2	52.5	+2 361.7	+57.3	2 419.0	0.968
	3–6	— 33	— 30.9									
	4–6	+833	+219.2									
3	3–5	—812	—221.4	—144.3	....	....	....	—144.3	+2 164.9	+57.3	2 222.2	0.889
5, 6	5–6	0	0	0	0	0	0	0	+2 309.2	+85.9	2 395.1	0.958
3′	3′–5	—812	—221.4	—541.9	....	....	....	—541.9	+1 767.3	+114.5	1 881.8	0.753
	4′–6	+625	+164.5									
	3′–6	+293	+274.6									
4′	3′–4′	—121	—145.2	....	—10.72	—10.72	—321.6	—687.1	+1 622.1	+114.5	1 736.6	0.695
1′	1′–3′	—634	—192.8	—561.5	....	....	....	—1 425.0	+884.2	+143.2	1 027.4	0.411
	2′–4′	+411	+145.1									
	1′–4′	+307	+228.3									
2′	1′–2′	+100	+ 73.8	....	— 7.97	—18.69	—560.7	—1 351.2	+958·0	+143.2	1 101.2	0.440
	0′–1′	—591	—255.0									
0′	0′–2′	+411	+145.1	—569.1	....	....	....	—2 481.0	—171.8	+171.8	0	0

* 1 kip denotes "kilo-pounds," or 1 000 lb.

Since the writer has modified the author's arrangement of the computations a brief summary of the procedure used will be given. To save space the individual products entering into the solution of the basic equations have not been recorded in Table 10.

In Column (5) are recorded the values of the vertical deflections of the panel reference points as computed by Equation (19). It should be noted that the deflections of Points *O* and *O′* are referred to Points 2 and 2′, respectively. In Column (6) are recorded the values of the rotations of each reference member due to the distortion of the given panel, as computed by Equation (22). Column (7) gives the total rotation of each reference member referred to Member 5–6. Column (8) gives the vertical deflection of each reference point due to rotation of the panel reference member. This value is given by Equation (23). The values in Column (9) represent the vertical deflection of each point relative to Point 5, assuming that Member 5–6 does not rotate. For example, the deflection of Panel Point 1 equals the deflection of Point 1 relative to Point 3, plus the deflection of Point 3 relative to Point 5, plus the deflection of Point 1 due to the rotation of the panel reference Member 3–4. This sum is given by $(-278.4 - 144.3 - 367.2) = -789.9$. Column (10) gives the values of the transposed deflections referred to Point O. It is seen from Column (10) that the deflection of Point O' relative to Point O is not zero, due to the assumption that Member 5–6 does not rotate. This means that the truss as a whole must be rotated clockwise about Point O through such an angle as to make the deflection of Point O' zero. This rotation is identical to that assumed in the Mohr diagram. The vertical deflection of each point for such a rotation will be given by the product of the horizontal distance of the point in question from Point O, multiplied by the rotation angle (in this case $\frac{171.8}{180}$). Column (12) gives the true deflection of each point, which is the sum of the values in Columns (10) and (11). In Column (13) the final values are converted to inches.

It is the writer's opinion that the foregoing procedure for applying the basic equations is easier to follow than that given by the author.

David A. Molitor,[15] M. Am. Soc. C. E. (by letter).[15a]—The method proposed by the author should scarcely be classified under such a broad title, because of its limited application to very simple truss forms.

In point of labor and clarity of presentation, the method of elastic weights affords equally simple solutions for the same cases, but with the advantage of being universally applicable to all kinds of structures, even to solid web-beams. The complications encountered in odd-shaped triangular truss systems without vertical members, are due to the more involved geometric relations, for which the new method becomes quite "out of bounds."

An exhaustive treatment of the method of elastic weights, based on Professor Otto Mohr's (1875) statement of the relation between deflection polygons and equilibrium polygons for simultaneous cases of loading, was presented by the writer in 1911.[16]

There is a decided danger that those who do not appraise the distinct limitations of this paper may misapply it, in view of the author's broad conclusion that the method "solves the problem of deflections in a simpler and more direct manner than other analytical methods."

Glenn L. Enke,[17] Jun. Am. Soc. C. E. (by letter).[17a]—The analytical method for solving for truss deflections presented by Mr. Shoemaker is an interesting one. It appears much superior to what is referred to in most text-books as the "method of internal work" wherein individual computations are made for any one panel point.

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[15] Structural Engr., Procurement Div., Public Works Branch, U. S. Treasury Dept., Washington, D. C.

[15a] Received by the Secretary February 11, 1936.

[16] "Kinetic Theory of Engineering Structures", by David A. Molitor, Chapter VII, 1911.

[17] Associate Bridge Designing Engr., Highway Div., State Dept. of Public Works, Sacramento, Calif.

[17a] Received by the Secretary February 3, 1936.

The writer prefers a graphical method utilizing Williot-Mohr diagrams to arrive at the vertical and horizontal displacement of all panel points simultaneously. An unsymmetrical or continuous structure is handled as easily as a symmetrical layout and throughout the process a rough idea of what is happening to any joint can be visualized; a valuable factor in eliminating blunders. By means of a drafting-machine, large-scale diagrams of sufficient accuracy are quickly prepared.

The writer's purpose in discussing this paper, however, is not to extol the merits of any particular method, but to point out the necessity of correctly applying results obtained by any method to the case in hand. Consideration of the various types of structures will be restricted to the modern types of highway and railway bridges.

Resident engineers on bridge construction from the writer's office have found, without exception, that actual truss deflections are uniformly less than the theoretical by 40 to 50%, and in setting header-boards for concrete surface finishing, they take this fact into consideration. Upon the accuracy of their estimate depends how level a roadway surface is obtained. This effect is a very common one, and is based upon a number of factors to be discussed subsequently. A specific instance is quoted: The Bridge Department of the State of California in 1935 completed a vertical lift structure over the Sacramento River, known as the M Street Bridge. The lift span is a symmetrical 202-ft, riveted, through truss span, 56.5 ft from center to center of trusses, with two unsymmetrical approach spans of 167.5 ft and 192.5 ft, respectively. A concrete deck with curbs 52 ft apart, over which four lanes of highway traffic and a single-track railroad pass, is supported by the usual system of floor-beams and stringers framed into each other and riveted throughout. Deflection diagrams were prepared for various conditions of dead load for each span. In erecting these trusses and pouring the concrete roadway slab, the resident engineer correctly estimated the deflection at about one-half the theoretical and set his grade line accordingly. A highly satisfactory profile was obtained, to the credit of the resident engineer. Many factors enter into a result such as the foregoing, as, for example: (1) Joint rigidity; (2) joint "slippage"; (3) direct stress taken by lateral bracing and steel floor systems; (4) using gross section to compute total elongation; and (5) reduced strains due to heavy lacing-bars, stay-plates and long gusset-plates.

Factor (1).—Probably the most important cause of reduced truss deflection is joint rigidity. In a completely riveted truss, every member is practically fixed at the ends (end restraint is not unknown in pin-connected members), and the secondary stresses set up by a nominal amount of deflection will produce a certain degree of resistance to further deflection in addition to a resistance already created by direct stresses. Analysis for secondary stresses can be made, and from these a set of panel-point deflections secured. The work involved, however, is very laborious, and the assumptions made are even broader than those used in the original computations. For purposes of analysis, the distance between intersections of gravity axes is considered as effective length. With this assumption, the moment of inertia of an individual member is decidedly not uniform throughout its length, although it is ordinarily considered so for convenience.

Factor (2).—Another kind of distortion brought about by joint rigidity and the resulting end moments induced in every member can be termed, "joint slippage", wherein the outer rivets in any connection deform laterally under secondary stress and permit a slight end rotation of the member. This effect is usually slight, and acts to relieve secondary stress.

Factor (3).—Despite every effort to avoid it, lateral bracing and steel floor systems do take a part of the direct stress originally intended by analysis for the chord members only, the percentage taken by each being nearly proportional to the relative areas, positions, and stiffness of connections. In a narrow structure with heavy wind-bracing and relatively light main members, a considerable portion of the direct stress can be resisted by lower chord-bracing.

Distribution of stress among chord members and floor systems is very uncertain. The writer recently conducted a few strain-gauge measurements on a lower chord of the lift span of the M Street Bridge. Available loading consisted of two electric freight engines, which, theoretically, would stress this lower chord in tension to 2 400 lb per sq in. Within the range of accuracy of the measuring equipment, this lower chord was stressed almost to this value. The live load was symmetrically placed, longitudinally and laterally, to avoid any eccentric loading. Strain-gauge readings were also taken on the lower flange of a roadway stringer parallel to, but a considerable distance away from, the railroad stringers directly supporting the live load, and, theoretically, not subject to railroad loading. This stringer is framed between floor-beams by means of light connection angles, but is rigidly attached to the concrete deck because of the field welding of the main slab reinforcement directly to the top flange of all roadway stringers. Longitudinal continuity of the slab across floor-beams is prevented by the presence of a cold joint along the center line of floor-beams; yet, under the same live load, it was able to register a tensile stress of about 500 lb per sq in. Although no definite conclusions can be drawn from this brief investigation, it is of interest to note the presence of direct stress in members distinctly not a part of the lower chord, nor directly connected thereto.

Factor (4).—A designer usually considers the gross section of all members in arriving at the total elongation, or strain. Actually, this is somewhat in error for built-up tension members, as directly across a line of rivets, the unit strain is greater than across a line between rivets. A more accurate method is to use a "weighted" cross-section wherein the rivet pitch and percentage of holes out, are taken into consideration.

Factor (5).—Another important effect is the presence of much detail material on the member, such as heavy lacing-bars, stay-plates, long gusset-plates, and portal and sway-frame gussets, all of considerable cross-section, which materially reduce the unit strain wherever they occur. This effect becomes most pronounced at the joints themselves. A haphazard allowance for these variables is impossible. A scientific analysis may be attempted for every truss member throughout its length to determine the variation in unit strain, but the numerous assumptions made while doing it scarcely warrant any high regard for the result.

In consideration of these variables, the writer wishes to conclude his discussion by warning against too much accuracy in determining truss deflections by any method, however excellent it may be, if the actual structure under consideration will behave quite differently from a theoretically pin-connected truss. As the majority of highway and railway structures to-day are completely riveted throughout, in addition to being well braced and comparatively rigid, the aforementioned facts deserve full consideration when applying truss deflections in the field.

Undoubtedly, the author is well aware of these limitations, but there are many who need a gentle reminder from time to time to prevent their interest in the mathematics of a solution from obliterating other, and often more important, phases of a problem.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SUCCESSIVE ELIMINATION OF UNKNOWNS IN THE SLOPE DEFLECTION METHOD

Discussion

BY MESSRS. C. A. WILLSON, PAUL ANDERSEN, AND R. W. STEWART

C. A. WILLSON,[6] M. AM. SOC. C. E. (by letter).[6a]—The slope deflection method of analyzing statically indeterminate structures is recognized by structural engineers as a valuable working tool. It is especially useful in the analysis of a complicated unsymmetrical framework. However, it is a fact that the solution of a large number of simultaneous equations can become very tiresome. Therefore, the method outlined by Professor Wilbur should be of real service to designing engineers who find it advisable to make complete analyses of structural frames.

The writer has found it convenient and sufficiently accurate in many cases to determine the bending moments in the members meeting at a particular joint by a consideration of sixteen members that radiate from it. In such a layout there are five unknown deflection angles, the one at the joint in question and four others. The writer has used the same idea advanced by Professor Wilbur of successive elimination of unknowns in the development of a simple solution.

The sixteen-member framework is preferable to the smaller frameworks frequently used because greater accuracy is obtained and because the effect of loading and unloading a number of members in the vicinity of the joint in question can be determined quickly.

Let Fig. 3 represent a part of a building framework composed of beams and columns of different lengths, moments of inertia, and loadings. Assume that bending moments are to be determined at Joint *O* because of the unsym-

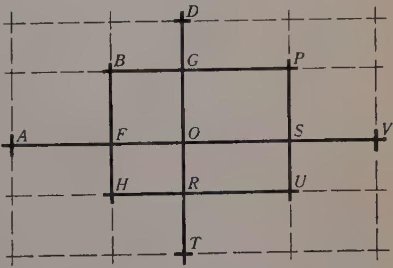

FIG. 3.

NOTE.—The paper by John B. Wilbur, Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order to bring the views expressed before all members for further discussion of the paper.

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[6a] Received by the Secretary January 8, 1936.

metrical conditions that surround that joint. Assume fixed-end terminals at a distance of two joints removed from Joint O in all directions. The resulting partial structure of sixteen members is indicated by heavy lines in Fig. 3. To summarize, fixed ends are assumed at A, B, D, P, V, U, T, and H, and bending moments are to be determined at Joint O.

The moments in members meeting at Joint R will be as follows:

$$M_{RT} = 4\ E\ \theta_R\ K_{RT} \qquad \text{(5)}$$

$$M_{RH} = 4\ E\ \theta_R\ K_{RH} + C_{RH} \qquad \text{(6)}$$

$$M_{RO} = 2\ E K_{OR}\,(2\ \theta_R + \theta_O) \qquad \text{(7)}$$

and,

$$M_{RU} = 4\ E\ \theta_R\ K_{RU} - C_{RU} \qquad \text{(8)}$$

Let $C_R = C_{RH} - C_{RU}$ and let $\Sigma\ K_R = K_{RT} + K_{RH} + K_{RO} + K_{RU}$. With $\Sigma\ M = 0$ at Joint R:

$$0 = 4\ E\ \theta_R\ \Sigma\ K_R + 2\ E\ \theta_O\ K_{OR} + C_R \qquad \text{(9)}$$

Then,

$$E\ \theta_R = -\frac{2\ E\ \theta_O\ K_{OR} + C_R}{4\ \Sigma\ K_R} \qquad \text{(10)}$$

Similarly,

$$E\ \theta_F = -\frac{2\ E\ \theta_O\ K_{OF} + C_F}{4\ \Sigma\ K_F} \qquad \text{(11)}$$

$$E\ \theta_G = -\frac{2\ E\ \theta_O\ K_{OG} + C_G}{4\ \Sigma\ K_G} \qquad \text{(12)}$$

and,

$$E\ \theta_S = -\frac{2\ E\ \theta_O\ K_{OS} + C_S}{4\ \Sigma\ K_S} \qquad \text{(13)}$$

The moments in members meeting at Joint O will be as follows:

$$M_{OR} = 2\ E K_{OR}\,(2\ \theta_O + \theta_R) = 4\ E\ \theta_O\ K_{OR} - \frac{E\ \theta_O\ (K_{OR})^2}{\Sigma\ K_R} - \frac{C_R\ K_{OR}}{2\ \Sigma\ K_R} \qquad \text{(14)}$$

$$M_{OF} = 2\ E K_{OF}\,(2\ \theta_O + \theta_F) + C_{OF} = 4\ E\ \theta_O\ K_{OF} - \frac{E\ \theta_O\ (K_{OF})^2}{\Sigma\ K_F} - \frac{C_F\ K_{OF}}{2\ \Sigma\ K_F} + C_{OF} \qquad \text{(15)}$$

$$M_{OG} = 2\ E K_{OG}\,(2\ \theta_O + \theta_G) = 4\ E\ \theta_O\ K_{OG} - \frac{E\ \theta_O\ (K_{OG})^2}{\Sigma\ K_G} - \frac{C_G\ K_{OG}}{2\ \Sigma\ K_G} \qquad \text{(16)}$$

and,

$$M_{OS} = 2\,E\,K_{OS}\,(2\,\theta_O + \theta_S) - C_{OS} = 4\,E\,\theta_O\,K_{OS} - \frac{E\,\theta_O\,(K_{OS})^2}{\Sigma\,K_S}$$
$$- \frac{C_S\;K_{OS}}{2\;\Sigma\,K_S} - C_{OS} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

With $\Sigma\,M = 0$ at Joint O and $\Sigma\,K_O = K_{OR} + K_{OF} + K_{OG} + K_{OS}$:

$$E_O\left[4\,\Sigma\,K_O - \frac{(K_{OR})^2}{\Sigma\,K_R} - \frac{(K_{OF})^2}{\Sigma\,K_F} - \frac{(K_{OG})^2}{\Sigma\,K_G} - \frac{(K_{OS})^2}{\Sigma\,K_S}\right]$$
$$= \frac{C_R\,K_{OR}}{2\;\Sigma\,K_R} + \frac{C_F\;K_{OF}}{2\;\Sigma\,K_F} + \frac{C_G\,K_{OG}}{2\;\Sigma\,K_G} + \frac{C_S\;K_{OS}}{2\;\Sigma\,K_S} - C_O \;\ldots\ldots\ldots (18)$$

and,

$$E\,\theta_O = \frac{\dfrac{C_R\,K_{OR}}{2\;\Sigma\,K_R} + \dfrac{C_F\;K_{OF}}{2\;\Sigma\,K_F} + \dfrac{C_G\,K_{OG}}{2\;\Sigma\,K_G} + \dfrac{C_S\;K_{OS}}{2\;\Sigma\,K_S} - C_O}{4\,\Sigma\,K_O - \dfrac{(K_{OR})^2}{\Sigma\,K_R} - \dfrac{(K_{OF})^2}{\Sigma\,K_F} - \dfrac{(K_{OG})^2}{\Sigma\,K_G} - \dfrac{(K_{OS})^2}{\Sigma\,K_S}} \ldots\ldots\ldots (19)$$

When the value of the deflection angle at Joint O is determined by Equation (19), the values of the other four unknown deflection angles (Equations (10) to (13)) can be found easily. Then, with known values of the five deflection angles, it is a simple matter to calculate the desired bending moments.

Paul Andersen,[7] Assoc. M. Am. Soc. C. E. (by letter).[7a]—A clear and interesting method of setting up the slope deflection equations necessary for the analysis of continuous structures, is contained in this paper. It should be noted, however, that the method of reducing the number of unknown quantities, as presented by the author, is one which suggests itself immediately after all the "joint" and "bent" equations are written, and as such the idea is not unknown to designers. Professor Wilbur's procedure is to eliminate unknown quantities as the equations are set up rather than afterward. In all cases the author's final equations can be derived by writing the total number of slope deflection relations and proceeding as outlined in the paper. Furthermore, designers who make use of the slope deflection method are not, as a rule, objecting to the large number of simultaneous equations; a careful tabulation and the use of two or three successive approximations will yield results which are very close to the actual rotations.

In the case of secondary stress analysis the author's method will involve more work than the usual process, the permanent and the temporary unknowns being the rotations of bottom chord joints and top chord joints, respectively.

Professor Wilbur's method can be used to advantage in dealing with a continuous beam; the slope deflection equations for this type of structure are so simple and few that much unnecessary writing is avoided by choosing one permanent unknown. It is interesting to note, however, that an application of the same principle to the theorem of three moments results in a still larger saving. Thus, referring to Fig. 1(*a*):

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[7] Balboa Heights, Canal Zone.

[7a] Received by the Secretary January 10, 1936.

$$2\,M_B\left(\frac{1}{K_{AB}}+\frac{1}{K_{BC}}\right)+\frac{M_C}{K_{BC}}=A_{BC}\ \ldots\ldots\ldots\ldots\ldots(20)$$

in which A_{BC} can be computed from the position and magnitude of the load, P, and K_{BC}. Express M_C in terms of M_B.

Applying the three-moment theorem to $B\,C\,D$:

$$\frac{M_B}{K_{BC}}+2\,M_C\left(\frac{1}{K_{BC}}+\frac{1}{K_{CD}}\right)+\frac{M_D}{K_{CD}}=A_{CB}\ldots\ldots\ldots\ldots(21)$$

in which A_{CB} likewise is known. After substituting for M_C the value in Equation (20), it is seen that from Equation (21), M_D is obtained in terms of M_B.

Applying the three-moment theorem to $C\,D\,E$:

$$\frac{M_C}{K_{CD}}+2\,M_D\left(\frac{1}{K_{CD}}+\frac{1}{K_{DE}}\right)=0\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

In Equation (22) substitute the values of M_C and M_D in terms of M_B, and solve for M_B.

The application of the theorem of three moments has the advantage of leading directly to the bending moments without computing first the changes of slope in which the designer is generally not interested.

R. W. Stewart,[8] M. Am. Soc. C. E. (by letter).[8a]—Systemization is the essence of slope deflection and this is recognized in the author's paper. However, it appears that one opportunity for better systemization has been overlooked; that is, improvement in the rules for signs in the use of the fixed-end moment constants.

The author's Equation (4) terminates with the term, $\pm\,C$. Authoritative treatises[9] would invert this double sign, making it $\mp\,C$. A study of several available expositions of these sign rules has convinced the writer that the use of a single minus sign for the fixed-end moments throughout all the standard equations will operate more satisfactorily than the double sign usually published, or the alternate plus and minus signs sometimes used.[10] Furthermore, the usual explanatory statements can be curtailed with greater resulting clarity. To illustrate this in an introductory manner the following solution of the author's Fig. 1 is offered, assigning numerical values to the load, its position, and the relative stiffnesses $\left(\frac{I}{L}\text{-values}\right)$ of the members. In this illustration the fixed-end moments are considered as the activating moments which a loaded span would exert on its end joints if these joints were fixed.

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[8a] Received by the Secretary January 21, 1936.

[9] *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.; also, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 75.

[10] "Modified Slope Deflection", by L. N. Evans.

The following rules of signs are offered as completely covering all conditions: (1) Clockwise rotations are taken as positive, the reverse as negative; (2) moments that tend to produce clockwise rotation are considered positive, the reverse negative; and (3) deflections that produce clockwise rotations are considered positive, the reverse negative. Any further statements the writer feels to be dangerous and likely to cause confusion, except that the reader may be asked to observe the obvious fact that a moment or a deflection at a point in a beam which tends to cause the beam on one side of it to rotate in a clockwise direction will tend to cause the part on the other side to rotate in a contra-clockwise direction. This is analogous to the shear changing from positive to negative under the load when a simple beam is loaded with a single concentrated load.

In the author's program for solving Fig. 1, the solution is carried from the left end of the beam to the right, thereby necessitating carrying the constants for the fixed-end moments through the equations for the two unloaded spans, *C D* and *D E*. An easier program will be to work from right to left, as is done in the following solution, thereby carrying these constants through the equations for a lesser number of spans. The fixed-end moments with proper signs and directional arrows are, in accordance with the foregoing conventionality, indicated on Fig. 4. The standard equations used are the same as those in the various existing texts except that in all standard equations the fixed-end moment constants are preceded by a minus sign only—not a double sign, nor alternating plus and minus signs. Relative stiffness or $\frac{I}{L}$ - values are indicated. Referring to Fig. 4:

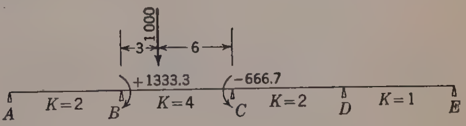

FIG. 4.

$$M_{ED} = 2\,(2\,\theta_E + \theta_D) = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

and, $\theta_D = -\,2\,\theta_E$;

$$M_{DE} + M_{DC} = 0 = 2\,(-\,4\,\theta_E + \theta_E) + 2 \times 2\,(-\,4\,\theta_E + \theta_C) \;\ldots (24)$$

and, $\theta_C = 5.5\,\theta_E$;

$$M_{CD} + M_{CB} = 0 = 2 \times 2\,(11\,\theta_E - 2\,\theta_E) + 2 \times 4\,(11\,\theta_E + \theta_B) + 666.7 \;\ldots (25)$$

and, $\theta_B = -\,15.5\,\theta_E - 83.33$;

$$M_{BC} + M_{BA} = 0 = 2 \times 4\,(-\,31\,\theta_E - 166.7 + 5.5\,\theta_E) \\ -\,1\,333.3 + 2 \times 2\,(-\,31\,\theta_E - 166.7 + \theta_A) \;\ldots\ldots\ldots\ldots (26)$$

and, $\theta_A = 82\,\theta_E + 833.3$;

$$M_{AB} = 0 = 2 \times 2\,(164\,\theta_E + 1\,666.7 - 15.5\,\theta_E - 83.33) \;\ldots\ldots\ldots (27)$$

and, $\theta_E = -\,10.67$.

Comments.—In Equation (25) the fixed-end moment has the plus sign because it is negative in Fig. 4 and is preceded by a minus sign in the standard equation, and − × − = +. The fixed-end moment of 1 333.3 is plus in Fig. 4 and minus in the standard equation making it + × − = − in Equation (26).

If the load acted upward, the signs in Fig. 4 would be reversed, thereby reversing the signs for these terms in Equations (25) and (26) and yielding $\theta_B = +10.67$. Likewise, the constant, H_{AB}, familiar to users of slope deflection, may always be preceded by the minus sign only in the standard equations provided it is delineated on the diagram in accordance with the convention stated.

The author's numerical solution of Fig. 2, using θ_A and the first-story *R*-value as the permanent unknowns, is of interest in that, apparently, it displays the full possibilities of slope deflection for this problem, which involves side-sway and is not so conveniently solved by end-moment distribution.

However, it is of interest and probably useful to show the following solution for this problem in which the phenomena of flexure are so treated as to eliminate the *R*-terms (deflection terms) which are necessary in the author's solution, and, otherwise, to simplify and reduce materially the work.

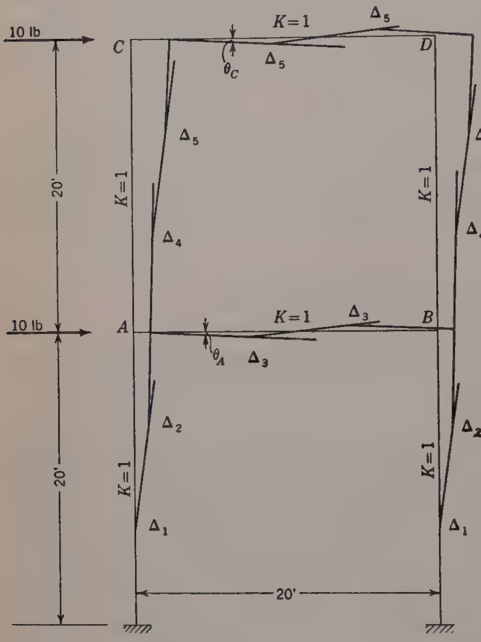

FIG. 5.

Referring to Fig. 5:

$(A)\ldots.2\,(2\,\Delta_1 + 2\,\Delta_2) = 20 \times 20$

and, $\Delta_1 = 100 - \Delta_2$;

$(B)\ldots.\theta_A = 100 - \Delta_2 - \Delta_2 \ldots\ldots\ldots\ldots\ldots\ldots\Delta_2 = 50 \quad - \quad 0.5 \quad \theta_A$

$(C)\ldots.2\,(2\,\Delta_4 + 2\,\Delta_5) = 10 \times 20$

and, $\Delta_5 = 50 - \Delta_4$;

$(D)\ldots.\theta_A = \frac{1}{3}\,(50 - \Delta_4) + 50 - \Delta_4 - \Delta_4 \ldots\ldots\Delta_4 = 28.6 - \quad 0.43\ \theta_A$

$(E)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots - \Delta_3 = \qquad - \quad 3.0 \quad \theta_A$

$(F)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \Sigma\,\Delta = 0 = 78.6 - \quad 3.93\ \theta_A$

and, $\theta_A = \dfrac{20}{E}$. The moment at the column base $= 2(100 - 50 + 10) = 120$.

Comments.—Since the stiffness factors of all members are equal and may be taken as unity, each moment will be double its corresponding Δ.

In Step (*A*), the sum of the internal moments is equated to the external moment for the first story; in Step (*B*), the column angles of the left lower column are added to obtain an expression for θ_A; Step (*C*) is the same as Step (*A*), except that it is for the second story; Step (*D*) is the same process as Step (*B*), except that it is for the left upper column; Step (*E*) is written directly from the geometry of the figure[11] and the terms are given negative signs because the obtuse supplement of Δ_3 indicates rotational activation opposite to Δ_2 and Δ_4; and Step (*F*) is the moment balance about Joint *A* which can be taken as the Δ-balance because all *K*-factors are equal.

If an independent check of the solution is desired, the traverse method with a graphical check, or an arithmetical angle summation check, will also be found to be simpler than slope deflection for Fig. 4. If the procedure for drawing the traverse heretofore set forth[12] is followed, it is easy to draw correctly the diagrams involved.

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[11] "An Improved Method of Finding Beam Deflections", *Civil Engineering*, February, 1934, p. 88; also "The Analysis of Continuous Structures by Traversing the Elastic Curves", *Proceedings*, Am. Soc. C. E., October, 1934 p. 1125.

[12] *Proceedings*, Am. Soc. C. E., September, 1935, p. 1071.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

REINFORCED CONCRETE MEMBERS UNDER DIRECT TENSION AND BENDING

Discussion

BY MESSRS. A. W. FISCHER, WILLIAM E. WILBUR, F. C. SNOW, AND RALPH E. BYRNE, JR.

A. W. FISCHER,[3] ESQ. (by letter).[3a]—The method of designing concrete members under direct tension and bending, as presented by the author, gives correct results. However, since values must be ascertained by using either Fig. 3 or Fig. 5 (which appear rather complicated) and generally must be interpolated, it seems that a more direct method can be used for solving such simple examples.

When e' is negative the author's method is as simple as any that the writer knows, but the suggested solution is not so simple when e' is positive. Consider Example 2 and Fig. 4 of the paper. Taking the center of moments about the centroid of the tension steel and solving for K, by Equation (20), $K = \frac{T e'}{b d^2} = \frac{16\,530 \times 10}{12 \times 10 \times 10} = 137.75$. Referring to a diagram in a standard textbook[4], with $n = 15$, $f_s = 18\,000$ lb per sq in., and $K = 137.75$; then, f_c is found to be equal to 796 lb per sq in. which practically agrees with the value derived by Mr. Gumensky. Furthermore, $k = \frac{n f_c}{f_s + n f_c} = \frac{796}{1\,200 + 796} = 0.3988$ (use 0.399), making $k\,d = 3.99$ in. The value of k can also be read directly from a diagram.[4]

Taking moments about the centroid of the compression area of the concrete (which is $\frac{1}{3}\,k\,d$ below the top of the beam in Fig. 4) the amount of tension steel required will be:

$$\frac{16\,530\left(e' + d - \frac{1}{3}\,k\,d\right)}{18\,000\left(d - \frac{1}{3}\,k\,d\right)} = \frac{16\,530\,(10 + 10 - 1.33)}{18\,000\,(10 - 1.33)} = 1.978 \text{ sq in. per lin ft,}$$

which corresponds with the results obtained by Equation (21).

NOTE.—The paper by D. B. Gumensky, Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

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[3a] Received by the Secretary December 27, 1935.

[4] See, for example, "Structural Members and Connections", by Hool and Kinne, p. 467.

If the stresses had been limited to $f_s = 18\,000$ lb per sq in., $n = 15$, and $f_c = 700$ lb per sq in., some compression steel would need to be added and, in that case, since the value of $K = 113.1$, the resisting moment in the concrete can be computed, and the remainder must be taken by compression steel.[5]

It seems to the writer that the foregoing procedure is more in line with that ordinarily used in beam design. The results can be derived very readily and all that is needed is a diagram giving the values of K and k, which are given in any standard textbook on reinforced concrete. All calculations have been made with a 20-in. slide-rule.

William E. Wilbur,[6] M. Am. Soc. C. E. (by letter).[6a]—It is a fact, frequently unnoticed by writers on structural design, that in determining the fiber stresses in a section of a beam to resist bending, the center of moments may be taken at any convenient point in the depth of the beam, and need not be at the neutral axis or at the geometrical axis. In a concrete beam, as the author shows in discussing his Case 2, it is advantageous to take the moment center at the location of the tensile steel. Using the nomenclature of Fig. 4, consider a beam subject to any bending moment, M, without direct stress, and take moments about the tensile steel; thus:

$$M = \frac{1}{2} f_c\, b\, k\, d \left(d - \frac{1}{3} k\, d \right) \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

which is the ordinary formula for beams subject to bending only. When the bending moment is that due to a force, T, with an eccentricity, e', the author's Equation (20) results. By carrying the foregoing analysis further, the author's process may be simplified considerably.

Referring again to Fig. 4, one may, without changing the conditions, add two equal and opposite forces, each equal to T, in the plane of the tensile steel. The external forces to be resisted are then a couple, of amount, $T\, e'$, and a direct stress, T, in the plane of the tensile steel. The moment, $T\, e'$, as was demonstrated previously, is resisted in the same manner as a moment due to vertical forces; and the beam may be designed for this moment in the same manner as is the ordinary rectangular beam. This determines the concrete stress, f_c, and the quantity of steel required for moment only. The force, T, acts in the line of the tensile steel, and must be resisted by an additional steel area, $\frac{T}{f_s}$. Then one may write:

$$A_s = p_1\, b\, d + \frac{T}{f_s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which p_1 is the percentage of steel required for moment only; and A_s is the total cross-sectional area of steel. This completes the design. Equation (25) may be readily derived from Equations (20) and (21), if $f_s\, p_1$ is substituted for its equivalent, $\frac{1}{2}\, f_c\, k$. The value of p_1 is determined readily from any of the ordinary diagrams for rectangular beams given in textbooks on concrete design.[7]

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[5] "Data on Design of Concrete", by A. W. Fischer, *Civil Engineering*, March, 1935, p. 180.

[6] Asst. Engr., Harrington & Cortelyou, Kansas City, Mo.

[6a] Received by the Secretary January 3, 1936.

The analysis of a beam subject to bending and direct tension then resolves into the following steps: (1) Determine the moment, $M = T\, e'$, in which e' is the distance from the tensile steel to the applied tension, T; (2) compute $K = \frac{M}{b\, d^2}$; (3) enter the diagram (for rectangular beam design) with the value of K, proceed to the desired value of f_s, and read directly the values of f_c and p_1; and (4) determine the total steel required, by Equation (25). This is seen to be much shorter than the author's method, and to require only the ordinary diagram for rectangular beam design found in textbooks.

To solve the example given by Mr. Gumensky: $T = 16\,530$ lb; $e' = 10$ in.; $n = 15$; $M = T\, e' = 165\,300$ in-lb; and, $K = \frac{M}{b\, d^2} = \frac{165\,300}{12 \times 10 \times 10} = 137.7$. Entering the diagram with this value of K, and reading at $f_s = 18\,000$, one finds at once, that $f_c = 800$, nearly, and $p_1 = 0.0088$. Then,

$$A_s = p_1\, b\, d = 0.0088 \times 120 = 1.056 \text{ sq in.}$$

$$+\frac{T}{f_s} = \frac{16\,530}{18\,000} = \underline{0.918}$$

$$1.974 \text{ sq in.}$$

The author finds a value of 1.92 sq in., the difference being no doubt due to differences in plotting and reading the diagrams.

An additional advantage of this method of analysis is that where the compressive stress is too high, compressive steel may be proportioned readily by the methods used in the design of ordinary rectangular beams reinforced for compression, proportioning for the bending moment, $T\, e'$.

The author treats of beams subject to bending and direct tension. As a corollary of this discussion, however, it may be noted that the foregoing analysis applies equally to the common case of beams under bending and direct compression, except that the area of the tensile steel is reduced, rather than increased, for the direct stress, the expression bcoming, $A_s = p_1\, b\, d - \frac{C}{f_s}$, in which C is the direct compression.

F. C. SNOW,[8] M. AM. SOC. C. E. (by letter).[8a]—No attempt to design for compressive and tensile reinforcement is made by Mr. Gumensky, and this is often necessary in members of fixed size, subject to bending and direct stress.

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[7] For example, "Structural Members and Connections", by Hool and Kinne, p. 467.

[8] Prof. of Civ. Eng., Georgia School of Technology, Atlanta, Ga.

[8a] Received by the Secretary January 25, 1936.

The following modifications of Mr. Gumensky's methods apply to reinforcement for tension only or for tension and compression if both are needed. Equation (10) can be written:

$$k_0 = \frac{1}{1 + \dfrac{f_s}{n\,f_c}} \quad \dotfill (26)$$

k_0 being the value of k which represents the limit of use of tensile steel alone, and demands the use of both compressive and tensile steel, if the f_c-stress is not to exceed a certain value.

If the value of f_c from Equation (10) is substituted in Equation (20):

$$\frac{T\,e'\,n}{f_s\,b\,d^2} = \frac{k^2\,(1 - \frac{1}{3}\,k)}{2\,(1 - k)} \quad \dotfill (27)$$

in which,

$$e' = \frac{M}{T} - d + \frac{d_t}{2} \quad \dotfill (28)$$

If the same value of f_c is substituted in Equation (21):

$$A_s = \frac{T}{f_s} + \left(\frac{k^2}{2\,(1 - k)}\right)\frac{b\,d}{n} \quad \dotfill (29)$$

TABLE 1.—CONSTANTS FOR USE IN REINFORCED CONCRETE BEAMS

k	$\frac{T\,e'\,n}{f_s\,b\,d^2}$	$\frac{k^2}{2\,(1-k)}$	k	$\frac{T\,e'\,n}{f_s\,b\,d^2}$	$\frac{k^2}{2\,(1-k)}$	k	$\frac{T\,e'\,n}{f_s\,b\,d^2}$	$\frac{k^2}{2\,(1-k)}$	k	$\frac{T\,e'\,n}{f_s\,b\,d^2}$	$\frac{k^2}{2\,(1-k)}$
0.10	0.0054	0.0055	0.27	0.0454	0.0499	0.44	0.1475	0.1729	0.60	0.3600	0.4500
0.11	0.0065	0.0068	0.28	0.0494	0.0544	0.45	0.1565	0.1841	0.61	0.3801	0.4771
0.12	0.0079	0.0082	0.29	0.0535	0.0592	0.46	0.1659	0.1959	0.62	0.4012	0.5058
0.13	0.0094	0.0098	0.30	0.0578	0.0643	0.47	0.1757	0.2084	0.63	0.4237	0.5364
0.14	0.0109	0.0114	0.31	0.0624	0.0696	0.48	0.1861	0.2215	0.64	0.4475	0.5689
0.15	0.0126	0.0132	0.32	0.0673	0.0753	0.49	0.1970	0.2354	0.65	0.4728	0.6036
0.16	0.0144	0.0152	0.33	0.0723	0.0813	0.50	0.2083	0.2500	0.66	0.5000	0.6406
0.17	0.0164	0.0174	0.34	0.0777	0.0876	0.51	0.2203	0.2654	0.67	0.5283	0.6802
0.18	0.0186	0.0198	0.35	0.0832	0.0942	0.52	0.2329	0.2817	0.68	0.5587	0.7225
0.19	0.0209	0.0222	0.36	0.0891	0.1013	0.53	0.2460	0.2988	0.69	0.5913	0.7679
0.20	0.0233	0.0250	0.37	0.0953	0.1087	0.54	0.2590	0.3159	0.70	0.6261	0.8167
0.21	0.0260	0.0279	0.38	0.1017	0.1165	0.55	0.2745	0.3361	0.71	0.6634	0.8692
0.22	0.0287	0.0310	0.39	0.1085	0.1247	0.56	0.2898	0.3564	0.72	0.7035	0.9257
0.23	0.0317	0.0344	0.40	0.1154	0.1333	0.57	0.3060	0.3778	0.73	0.7467	0.9869
0.24	0.0349	0.0379	0.41	0.1230	0.1425	0.58	0.3230	0.4004	0.74	0.7933	1.0531
0.25	0.0382	0.0417	0.42	0.1358	0.1521	0.59	0.3410	0.4245	0.75	0.8438	1.1250
0.26	0.0417	0.0457	0.43	0.1390	0.1622						

In Table 1 various values of k are computed in terms of $\dfrac{T\,e'\,n}{f_s\,b\,d^2}$ and $\dfrac{k^2}{2\,(1-k)}$.

Use of Table 1 for Tensile Steel Only.—In this case k_0 is computed by Equation (26) and $\dfrac{T\,e'\,n}{f_s\,b\,d^2}$ is computed from the data of the beam. Then, k is taken from Table 1; if it is less than, or equal to, k_0 only tensile steel is needed. Area A_s is computed by Equation (29), taking the value of $\dfrac{k^2}{2(1-k)}$ from Table 1.

Applying the foregoing modifications to Example 1 of the paper, and assuming that f_c does not exceed 800 lb per sq in., and that f_s does not exceed 18 000 lb per sq in.: By Equation (26), $k_0 = 0.40$; by Equation (28), $e' = 10$ in.; and by Equation (27), $\dfrac{T\,e'\,n}{f_s\,b\,d^2} = 0.1148$. From Table 1, therefore, $k = 0.398$; and $\dfrac{k^2}{2(1-k)} = 0.1316$ by interpolation.

Since k is less than k_0 only tensile steel is needed and by Equation (29), $A_s = 1.97$ sq in.

Use of Table 1 for Compressive and Tensile Steel.—Assume that $M = 25\,000$ lb-ft, and that, otherwise, values are the same as in the foregoing case; then, by Equation (28), $e' = 14.14$ in.; and by Equation (27), $\dfrac{T\,e'\,n}{f_s\,b\,d^2} = 0.1623$. From Table 1, $k = 0.456$; and $\dfrac{k^2}{2(1-k)} = 0.1911$. Since k is greater than k_0 both compressive and tensile steel are needed.

If the reinforcement in the compressive side of a beam is, A'_s, placed a distance of d' in. in from the extreme fiber, the stress carried by this steel is,

$$A'_s\,f_c\,(n-1)\left(\frac{k\,d-d'}{k\,d}\right)\ldots\ldots\ldots\ldots(30)$$

Equation (20) becomes,

$$T\,e' = \frac{1}{2}f_c\,k\,b\,d\left(d-\frac{k\,d}{3}\right) + A'_s\,f_c\,(n-1)\left(\frac{k\,d-d'}{k\,d}\right)(d-d')\,..(31)$$

and Equation (21) becomes,

$$T - A_s\,f_s + \frac{1}{2}\,f_c\,k\,b\,d + A'_s\,(n-1)\,f_c\left(\frac{k\,d-d'}{k\,d}\right) = 0\ \ldots..(32)$$

Solving Equation (31) for A'_s,

$$A'_s = \frac{T\,e' - \dfrac{1}{2}\,f_c\,k\,b\,d\left(d-\dfrac{k\,d}{3}\right)}{(n-1)\,f_c\,\dfrac{(k\,d-d')}{k\,d}\,(d-d')}\ldots\ldots\ldots\ldots(33)$$

Solving Equation (32) for A_s,

$$A_s = \frac{T}{f_s} + \frac{\dfrac{k^2}{2(1-k)}\left(\dfrac{b\,d}{n}\right)}{f_s} + \frac{A'_s\,(n-1)\left(\dfrac{k\,d-d'}{k\,d}\right)}{f_s}\ldots\ldots(34)$$

Substituting f_s in terms of f_c from Equation (10) in Equation (34):

$$A_s = \frac{T}{f_s} + \frac{k^2}{2(1-k)}\left(\frac{b\,d}{n}\right) + A'_s\,(n-1)\left(\frac{k\,d-d'}{n\,(1-k)\,d}\right)\ldots.(35)$$

If the value of A'_s from Equation (33) is substituted in Equation (34),

$$A_s = \frac{T}{f_s} + \frac{k^2}{2\,(1-k)}\left(\frac{b\,d}{n}\right) + \frac{T\,e'}{f_s\,(d-d')} - \frac{k^2\left(1-\frac{k}{3}\right)}{2\,(1-k)}\left(\frac{b\,d^2}{n\,(d-d')}\right) \;\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Substituting f_c in terms of f_s and k from Equation (10):

$$A_s = \frac{T}{f_s} + \frac{k^2}{2\,(1-k)}\left(\frac{b\,d}{n}\right) + \frac{T\,e'}{f_s\,(d-d')} - \frac{k^2\left(1-\frac{k}{3}\right)}{2\,(1-k)}\left(\frac{b\,d^2}{n\,(d-d')}\right) \;\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

but $\dfrac{k^2\left(1-\frac{k}{3}\right)}{2\,(1-k)} = \dfrac{T\,e'\,n}{f_s\,b\,d^2}$ by Equation (27), so that Equation (37) is the same as Equation (29). In other words, the latter can be used to compute the tensile steel whether or not compressive steel is needed.

The foregoing substitution from Equation (27) involves the assumption that f_c can be made large enough so that no compressive steel is needed—namely, that $f_c = f_s\left(\dfrac{k}{n\,(1-k)}\right)$, k in this case being greater than k_o, making f_c greater than the allowable working stress. This statement indicates directly that Equation (29) applies both to tensile and compressive reinforcement conditions, but the foregoing proof is given as a further check.

Equation (35) will be used to compute A'_s and, since f_c cannot be increased beyond its allowable value, k_o must be used instead of k. By Equation (35), substituting the value of A_s from Equation (29):

$$\frac{T}{f_s} + \left(\frac{k^2}{2\,(1-k)}\right)\frac{b\,d}{n} = \frac{T}{f_s} + \left(\frac{k_o^2}{2\,(1-k_o)}\right)\frac{b\,d}{n} + A'_s\,(n-1)\left(\frac{k_o\,d-d'}{n\,(1-k_o)\,d}\right)$$

from which,

$$A'_s = \left[\frac{k^2}{2\,(1-k)} - \frac{k_o^2}{2\,(1-k_o)}\right]\left(\frac{(1-k_o)\,b\,d^2}{(n-1)\,(k_o\,d-d')}\right) \ldots\ldots (38)$$

Applying Equation (29) to the example for $k = 0.456$, Table 1 shows that $\dfrac{k^2}{2\,(1-k)} = 0.1911$; and for $k_o = 0.400$, Table 1 shows that $\dfrac{k_o^2}{2\,(1-k_o)} = 0.1333$.

By Equation (29) $A_s = 2.45$ sq in.; and by Equation (38), $A'_s = 1.49$ sq in. In this case the compressive steel is not excessive, but suppose that $M = 21\,400$ ft-lb, $T = 3\,915$ lb, $b = 12$ in., and $d = 8$ in.; then $e' = 68.6$ in.; and

$\frac{T\,e'\,n}{f_s\,b\,d^2} = 0.2913$. From Table 1, $k = 0.561$; and $\frac{k^2}{2(1-k)} = 0.3584$. With $k_0 = 0.4$, as before, $\frac{k^2_0}{2(1-k_0)} = 0.1333$, from Table 1. Then, by Equation (38), $A'_s = 6.17$ sq in. Should this steel area be considered excessive, it can be decreased by reducing f_s, keeping f_c constant. Assuming that f_s is 11 500: $\frac{T\,e'\,n}{f_s\,b\,d^2} = 0.4561$ (from the beam data). From Table 1, $k = 0.645$; and $\frac{k^2}{2(1-k)} = 0.5838$. By Equation (26), $k_0 = 0.510$; and by Table 1, $\frac{k^2_0}{2(1-k_0)} = 0.2654$. By Equation (38), $A'_s = 4.11$ sq in.; and by Equation (29), $A_s = 4.08$ sq in. This solution makes $A_s = A'_s$ and required three trials for varying values of f_s.

If f_s in terms of f_c from Equation (10), and A'_s for A_s, are substituted in Equation (35), and after solving for A'_s the resulting expression is equated to A'_s from Equation (33), a cubic equation in k_0 results, from which k_0 for $A_s = A'_s$ can be computed for any value of f_c. It is just as easy, however, to make three or four trial computations like the one just completed for different values of f_s until $A_s = A'_s$, as it is to solve the cubic equation in terms of k_0, should it be desired to make $A_s = A'_s$.